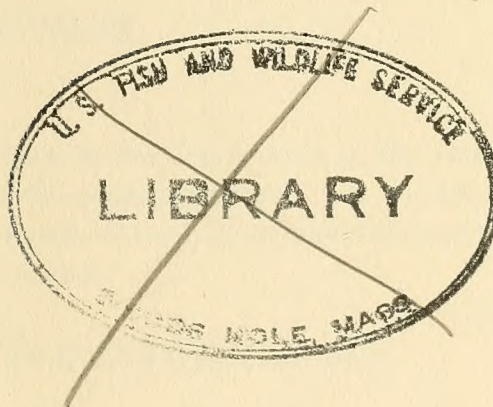




A MOBILE TEMPERATURE SENSING ARRAY FOR THE OCEAN

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THE PROBLEM

Investigate subsurface hydrodynamic phenomena in the sea; specifically, study internal waves by providing a mobile apparatus for the measurement of internal waves.

RESULTS

1. A submarine-mounted vertical array of thermistors for temperature measurements at several levels was designed, constructed, and tested.
2. The precision (repeatability), using probable error as criterion, is $\pm 0.0032^{\circ}\text{C}$. The accuracy (absolute) is $\pm 0.003^{\circ}\text{C}$ over the usable range.
3. Data output is in digital form.
4. The digital data are reduced to give depth of isotherms (i. e., internal waves) as a function of position along the track. Techniques for accomplishing this with a high speed computer have been devised and tested.

RECOMMENDATIONS

Accord recognition to the importance of the submarine as a working platform for the oceanographer. Use a submarine, especially during the season of well-developed thermoclines, for the study of internal waves.

ADMINISTRATIVE INFORMATION

This work was done under S-R011 01 01, Task 0401 (NEL L4-3) by members of the Signal Propagation Division. It commenced in July 1959, and was completed in November 1961. This

report, which was approved for publication 27 July 1962, complements the internal waves study of others in the Marine Environment Studies Branch.

Acknowledgement of assistance given by the Field Engineering Branch is gratefully made.

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INTRODUCTION

A heavy fluid with a free surface, having a density variation only in the vertical direction (for the undisturbed medium), is subject to two basically different types of gravity wave excitation: a surface wave, and internal waves. Internal waves are characterized by having, for every mode, the maximum vertical particle displacement below the free surface.

Most of the ocean for most of the year consists of two parts. The first part is the surface layer consisting of well mixed, nearly isothermal water; the second part is the lower, more dense water in which the temperature and density vary slowly. Separating these two parts of the ocean is the (seasonal) thermocline region in which the vertical temperature gradient is sizable. Since the horizontal salinity gradient is very slight in this region, isothermal surfaces nearly coincide with isopycnal surfaces. Thus, an obvious way to measure internal waves is to measure the vertical temperature structure in the vicinity of the thermocline.

A vertical string of thermistor temperature sensors for detection of the passage of internal waves was first used by Ufford; * such arrays are commonly used today. This technique is fine in shallow water, and can even be used in deep water through the utilization of deep-sea mooring devices. When it is necessary to survey an area, an instrument on a moving platform is desirable. One such scheme is the towed chain, which is used to measure the thermal microstructure of the sea.** The one described in reference 2 and also those devised since then cover an extensive range of depth with sizable spacing between sensor elements. Although provision is made for determining the depth of tow, this is usually done at only one point; the depth of individual sensor elements can be inferred from the shape of the chain.

*Reference 1, page 55

**Reference 2, page 55

Since the temperature change (i.e., signal) for a given vertical displacement* in the thermocline is proportional to the vertical temperature gradient, this region is a desirable one in which to operate. To measure the vertical displacement of isotherms in the thermocline, fairly close spacing of sensor elements, precisely located, is required. Close, precise spacing can be achieved with the moving strut internal wave recorder described here. The results obtained in a successive determination of such vertical displacement of isotherms can be portrayed as a function of time (fig. 13), the curves representing internal waves.

APPARATUS

A vertical array of temperature sensors was achieved by installing a number of normalized thermistors, equally spaced, in the leading edge of a vertical strut. The strut was mounted at the bow of a submarine (see fig. 1A for a low drag strut, and fig. 1B for tower). The thermistors operated on linearizing bridges, the outputs of which were recorded sequentially. Once each sequence, the output of a pressure transducer (designed to determine the depth of the strut) was also recorded as indicated schematically in figure 2.

THE STRUT

The first strut used was wooden, and 18 feet in length. Its leading edge was semicircular, with radius of 3 inches. For ease of construction, the strut was faired with a straight-sided wedge such that the chord was 18 inches. At 6 knots, the coefficient of drag was estimated at 0.16, and thus the estimated total drag on the strut was 230 pounds at 6 knots. The foot of the strut was mounted in a steel boot, and the strut held in place by a number of

*The vertical displacement as a function of depth is also probably a maximum at the thermocline for the type of problems the authors are interested in.



Figure 1A. Low drag strut array mounted on USS BAYA



Figure 1B. Tower array mounted on USS SEA DEVIL

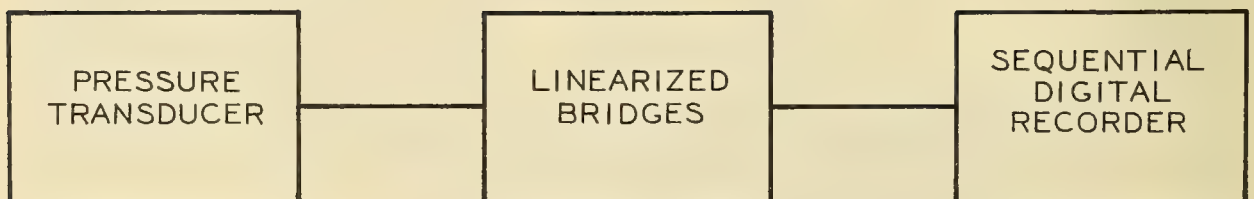


Figure 2. System block diagram

smaller struts (fig. 1A). The temperature sensors were placed 2-1/2 feet apart, starting 4 inches from the top. The shoulder of the sleeve of the sensor element was flush with the leading edge, and the thermistor bead extended about 1/8 inch into the fluid stream.

The strut currently in use (fig. 1B) is made of three sections, each 10 feet long, of a commercial antenna tower. Each section is triangular in cross-section, having three 1-inch tubular vertical pipes braced by flat straps. The tower is suitably guyed laterally, as well as fore and aft. At 6 knots, the coefficient of drag for the circular tubes was 1.2 and for the straps it was estimated to be 1.5; thus the estimated total drag on the strut was 1675 pounds at 6 knots. There are 34 thermistor beads in the vertical array. Of these, 26 are used with the digital data system; they are spaced at 1-foot intervals. Five are used as in the original array, except that they are spaced at 6-foot intervals. The remaining three beads all have fast thermal response, and are used for continuous temperature measurements. Two of these beads are located near the middle of the tower while the third is located at the top. In addition, there is a fixed resistor which is mounted at the foot of the tower.

It is clear that the motion of the platform through the water causes an essentially time-independent distortion of the stream-lines about the bow. This leads to a distortion of the time-dependent vertical temperature structure, and the shape of the internal wave is modified. Since the distortion is most severe near the hull, the lowest beads are mounted some distance above the hull, and of the several isotherm signals obtained, the ones intercepted by the upper part of the strut are preferred.

TEMPERATURE INDICATING SYSTEM

Thermistors were chosen as temperature sensor elements because of their high sensitivity and simplicity. Thermistors have a large negative temperature coefficient of resistance, and the resistance is an exponential function of temperature. Commercially available thermistors do not have the same resistance vs.

temperature characteristics (typical variations are ± 20 percent in nominal resistance at 25°C), and hence are not directly interchangeable. Since the feature of interchangeability in sensor units is very desirable, a method was worked out (see Appendix A) for determination of the optimum values of two resistors (fig. 1, Appendix A), one in series and one in parallel with the thermistor, in order to obtain the same resistance vs. temperature calibration curve for all sensor units. This process is called normalization. The thermistors used were Veco 32A1,* which is the bead type with a glass probe covering. They are 2 inches in length, have adjacent leads, and have a nominal resistance at 25°C of 2000 ohms.

Small watertight housings were constructed to contain the thermistor and normalizing resistors (fig. 3). The normalizing resistors were wire wound on ceramic bobbins and have radial leads, allowing the thermistor to be passed through the bobbin holes. These components together with fiber spacers and a wire

*Victory Engineering Corp., Springfield Road, Union, New Jersey

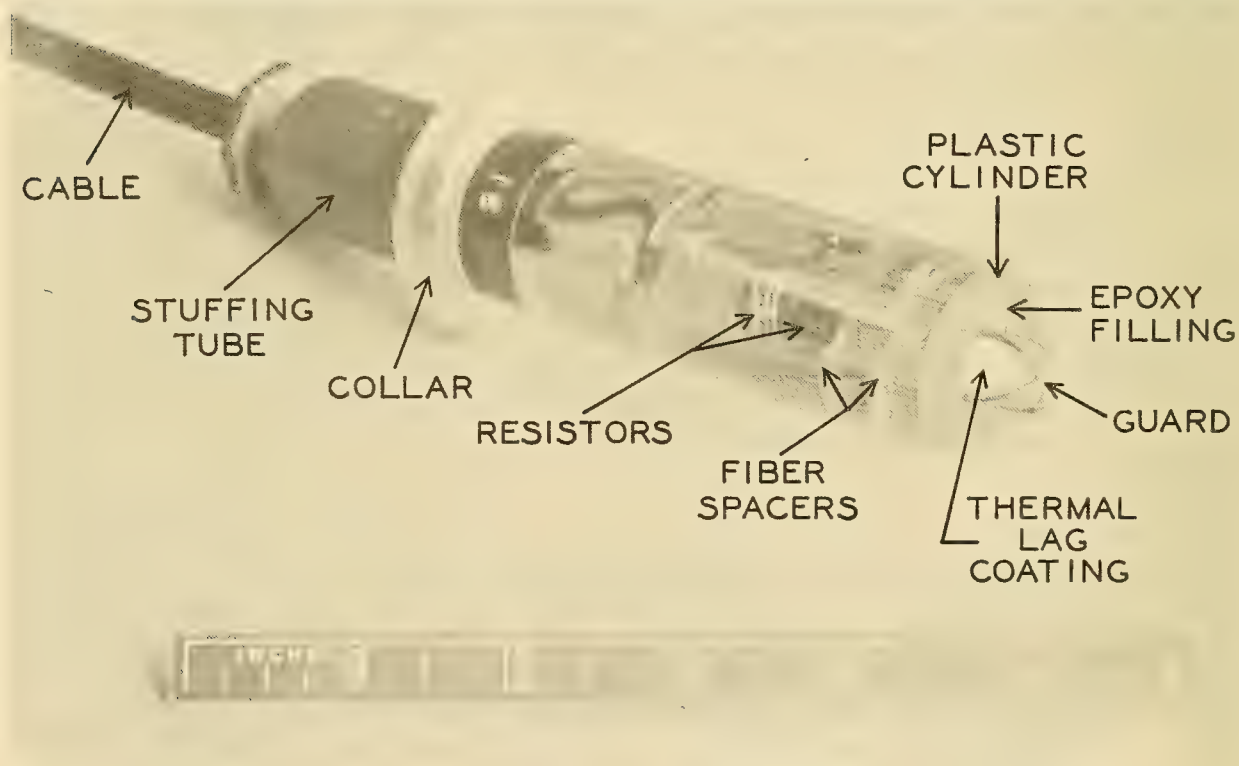


Figure 3. Thermistor assembly

guard formed a subassembly which was inserted in a plastic tube and collar after connections to the main cable were made. The plastic tube was threaded to fit a standard size A, brass stuffing tube that provides a watertight seal between the cable and the unit. The tube was then filled with epoxy resin, leaving the tip of the thermistor bead extending approximately 1/8 inch.

The thermal time constant of the thermistor thus encapsulated was approximately 1/2 second. Since the temperature readings were to be taken at intervals of from 10 seconds to 1 minute, "aliasing" of the temperature signal could occur unless a thermal lag were introduced. For this reason, the thermal time constant was increased to about 16 seconds by applying several coats of Armstrong A-2 epoxy resin. Measurement of the time constant during this process was done by rapidly immersing the thermistor unit in a homogeneous water bath that was maintained several degrees below room temperature. The thermistor unit being tested was used with a bridge circuit (to be described in the next section) whose output was fed to a Minneapolis-Honeywell potentiometer strip chart recorder with a full scale pen travel of 1/4 second. From this temperature trace as a function of time, the time constant was easily determined (fig. 4).

Since the temperature of the probes was to be recorded electrically, a bridge circuit was needed to provide a dc voltage output proportional to the temperature. The basic form of the bridge that has been used, called an asymmetric half-bridge, is shown in fig. 5B. In order to obtain a linear voltage output, e_o , with respect to temperature, the value of r , is of prime importance. For a given set of normalized thermistor sensors there is an optimum value of this resistor, r , for linear output. A method for obtaining this optimum value is described in Appendix II. In order to adjust the bridge to measure a certain temperature range, both zero and span controls are used. Thus, the bridge can be balanced for zero output voltage at any desired temperature, and the span can then be adjusted to make the full scale temperature compatible with the limits of the recording system.

In the equivalent circuit (fig. 5A) shown the ratio of E_2/E_1 adjusts the zero or balance point of the bridge, while the sum,

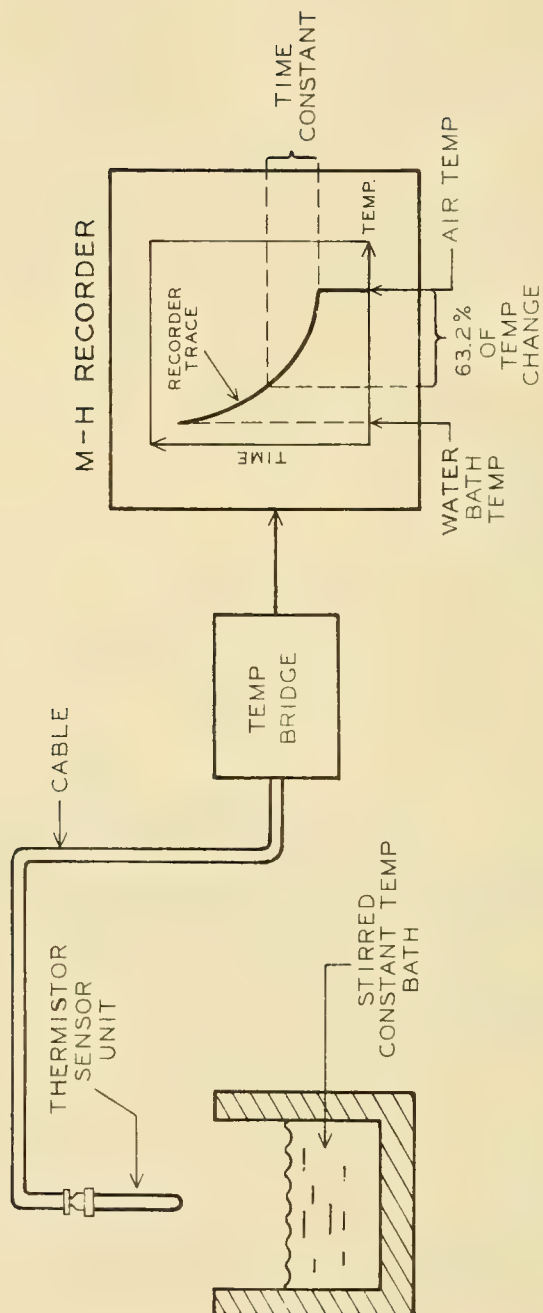


Figure 4. Apparatus for finding the time constant of temperature sensor

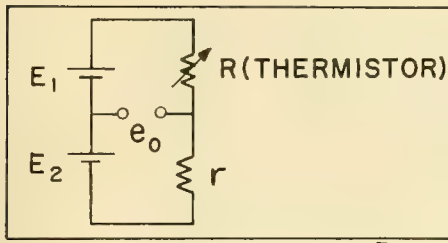


FIG. 5 a

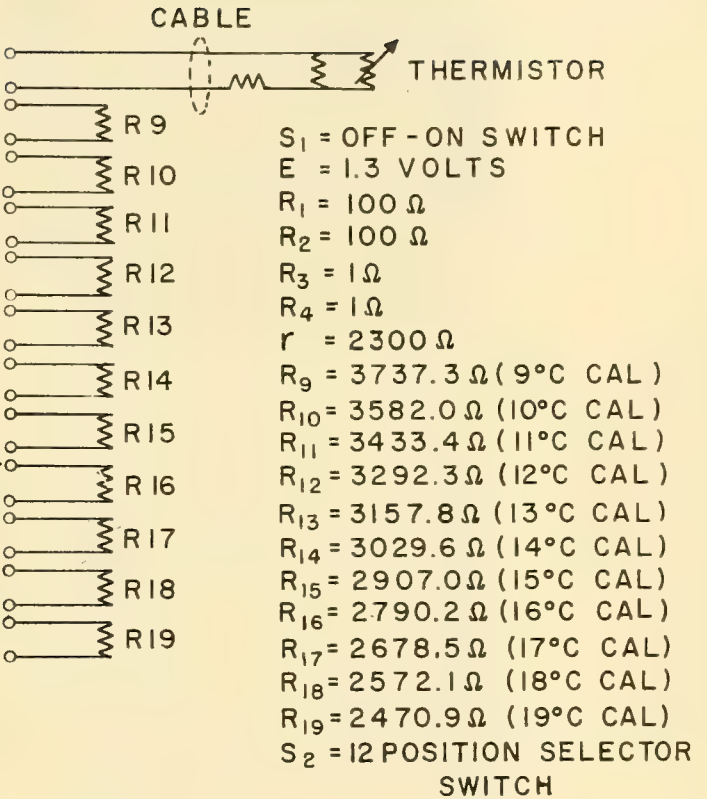
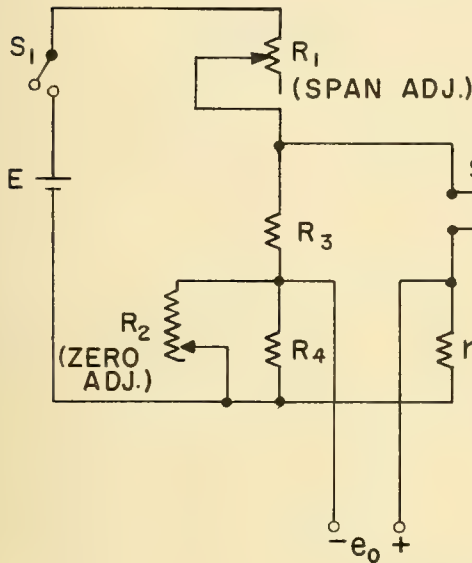


FIG. 5 b

Figure 5. Basic temperature bridge

$E_1 + E_2$, adjusts the span. Thus, for zero output at temperature T_x , with corresponding thermistor resistance R_x ,

$$\frac{E_2}{E_1} = \frac{r}{R_x}$$

In practice, the batteries are replaced by a low resistance voltage divider network. The zero is adjusted by the resistor, R_2 . Also, the power dissipation of the thermistor in the medium in which it is to be used should be considered, i.e., the maximum current through the thermistor should be small enough so as to produce negligible self-heating. In the bridge circuit discussed above, the maximum allowable current was held to 50 microamperes, by the use of resistor, R_1 . Adjustment of R_1 does not alter the balance point of the bridge but only the sensitivity or span. Since it

was desired to have a temperature range of 9 - 19°C, the bridge was first zeroed to give zero voltage output at 9°C and then the span control was adjusted to give approximately a 20 millivolt output at 19°C. The adjustment of the zero and full scale values are made by inserting, in place of the thermistor, a wire-wound resistor having resistance first equal to that of the mean thermistor values at 9°C, and then equal to that of the mean thermistor values at 19°C. A means of field-checking the operation of the bridge is provided by inserting wire-wound resistors with resistance corresponding to that of the thermistor at intervals of 1°C.

Mercury batteries were chosen as power supplies because of their voltage stability as a function of time after the initial aging. Mallory type RM-42R mercury batteries were used in all of the temperature bridges. Only one basic bridge has been discussed so far. Where the temperatures of several sensors were to be recorded on one recorder system by sequential sampling, several bridges were used in parallel, with one common power supply. Thus, the positive output for each additional bridge (consisting of one sensor unit and one linearizing resistor, r) added in parallel, can be referred to one negative output terminal, common to all bridges. The values of R_3 and R_4 are made small so as to provide essentially a zero impedance excitation potential, thus reducing any interaction between bridges.

PRESSURE INDICATING SYSTEM

It was necessary to record depth variations as a function of time along with the temperature records, so that the depth variations could be suitably compensated at a later time (see section entitled Data Reduction). Since the depth desired was that of the temperature sensors, the ideal location of the pressure transducer inside the hull would be directly under the strut or tower containing the temperature sensors. In this location the errors in measuring depth of the sensors, with the submarine not at zero bubble,* are reduced to a minimum. By mounting the pressure

*Submarine at zero bubble means that the longitudinal axis is horizontal.

transducer inside the hull, simplicity and reliability are obtained since no watertight housings, etc., are needed. The transducer is simply connected to the nearest sea chest by means of a high pressure line; the electrical leads are connected to the associated control and recording system. The pressure at the sea chest has a quasi-static component, and a fluctuating component due to the surface wave action. The latter pressure drops off exponentially with distance from the free surface and in usual operation is sufficiently attenuated. Near the surface, these fluctuating components would have to be filtered out to prevent aliasing when the pressure reading is sampled at intervals equal to, or longer than, half-surface-wave periods.

A Statham Model PA208TC unbonded strain gage pressure transducer was used.* This was an absolute gage with a range of 0-500 psia and a combined nonlinearity and hysteresis of less than ± 0.75 percent of full scale. The basic bridge circuit used with the pressure transducer, recommended by Statham, is shown in fig. 6. In this circuit there is only one adjustment necessary and this is the zero adjustment. Resistor, R_1 , is the coarse zero adjustment and the combination of R_3 and R'_3 is the fine adjustment. In practice, this adjustment was used in order to make this bridge coincide with the reading obtained on the ship's control room bourdon gage, with the submarine at zero bubble. Thus, comparisons continuing throughout the sea test provided one method of calibration. Another type of calibration consists of inserting the resistors R_{C1} through R_{C5} one at a time, by means of corresponding momentary switches S_2 through S_6 .

From the manufacturer's calibration data for the transducer, it is possible to calculate the value of the calibrate resistor which corresponds to a given pressure interval, and thus a known depth interval. For example, if the value of resistor, R_{C1} , corresponds to a 25 foot interval, then, when it is inserted in the circuit, this will produce a change in voltage output, e_0 , corresponding to 25 feet. It should be emphasized that this change is superimposed on the existing pressure signal. The depth calibration intervals

*Statham Laboratories, Inc., 12401 W. Olympic Blvd., Los Angeles 64, California

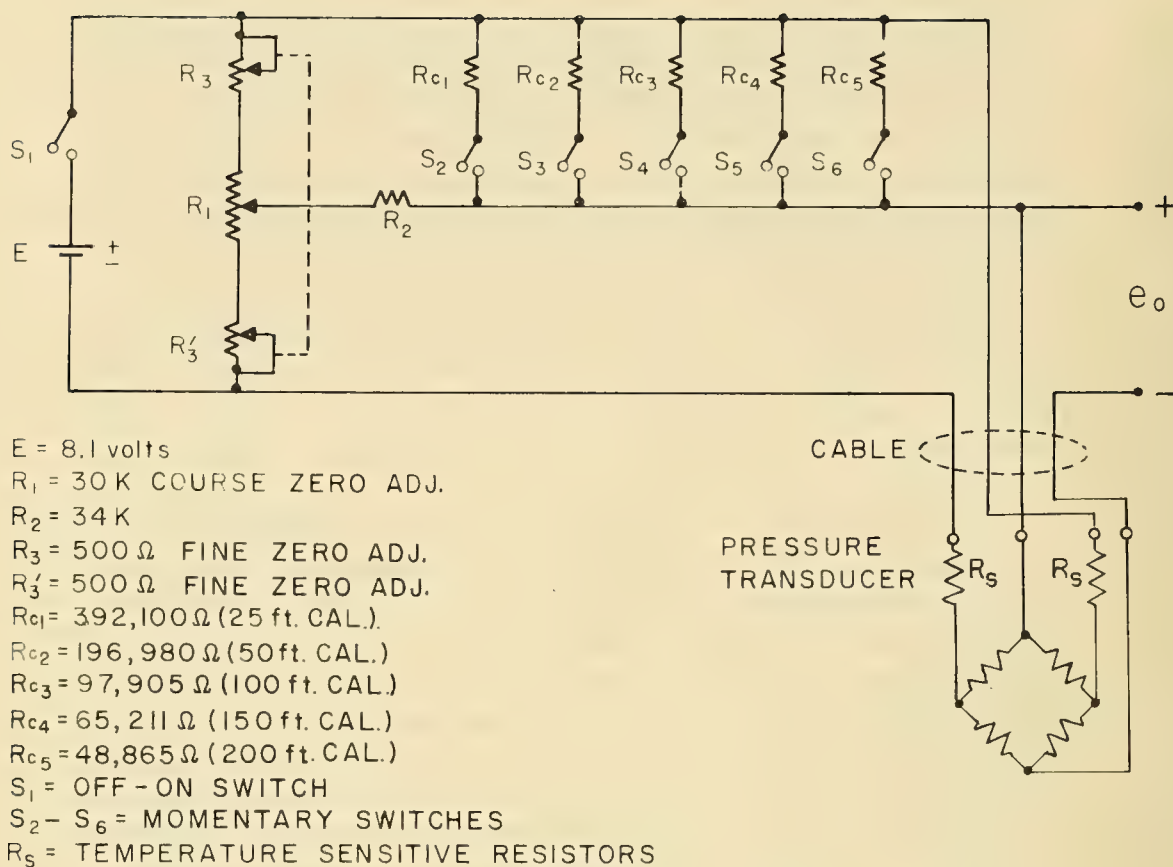


Figure 6. Depth bridge for data system

chosen are shown in the diagram for the remaining calibrate resistors.

A convenient laboratory method for determining the voltage output, e_o , as a function of depth is to expose the transducer to atmospheric pressure, which provides a sufficiently stable reference point, zero the bridge, and then measure e_o as the individual calibrate resistors are inserted. These resistors can also be used to make calibrations for sensitivity in the field; however, these calibrations should be made when the ship is at zero bubble and maintaining depth well.

Mercury batteries (Mallory type TR-136R) were also used in the depth circuits; they had a no-load voltage of 8.10 volts. A modification (not shown) of this basic bridge was made for use with

strip chart recorders that had fixed ranges. The output was attenuated so as to make a range of 100 feet cover the calibrated part of the recorder. Also, a bucking voltage was added to the positive output in order to use several overlapping ranges, each of 100 feet.

DATA RECORDING SYSTEMS

At the present time there are two independent data recording systems, one of which is the visual monitor and data recording system (fig. 7). This system is the more simple in form and was initially used as the only recording system. Temperatures from five thermally lagged thermistors were recorded on the first five channels of a six-channel strip chart recorder while the sixth channel recorded depth readings from the pressure transducer. The recorder used was a Minneapolis Honeywell, universal multipoint, potentiometer type, model 153 x 82, with a 0-2 millivolt range.*

*Minneapolis-Honeywell Regulator Co., Wayne and Windwin Avenues, Philadelphia 44, Pa.

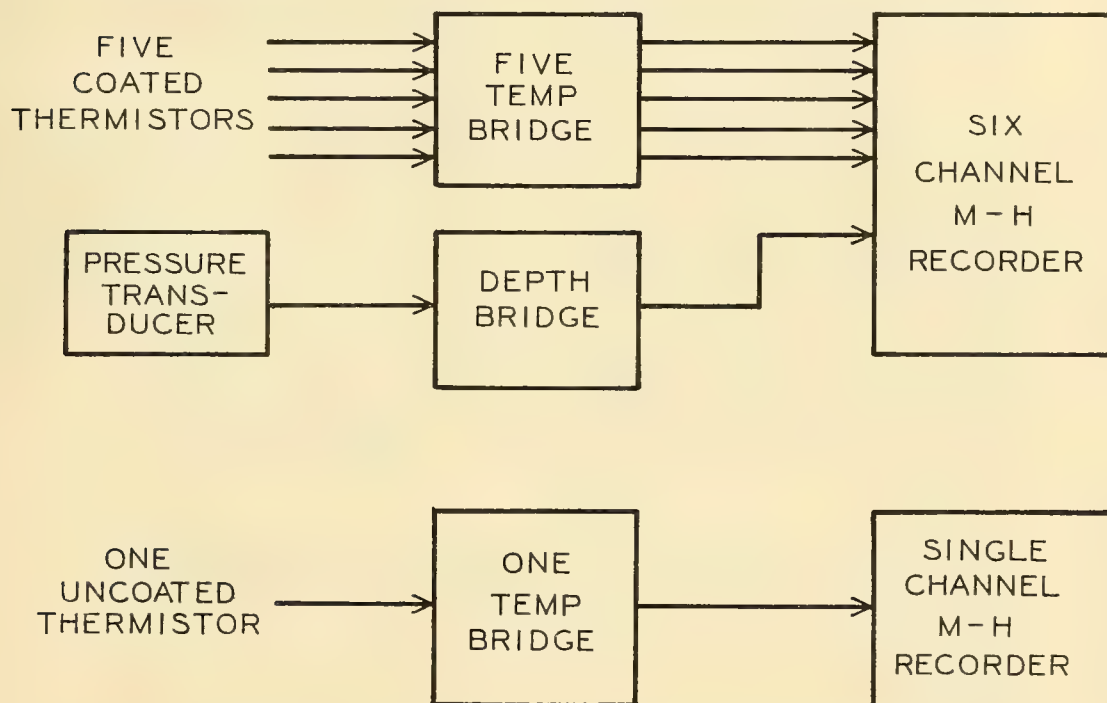


Figure 7. Visual monitor and recording system

The temperature bridges were adjusted so as to read from 9°C to 19°C, and the pressure bridge adjusted so as to produce the following five ranges which are given in feet: 0-100, 50-150, 100-200, 150-250, and 200-300. Another strip chart recorder was used to record continuously the relatively fast temperature fluctuations of one uncoated thermistor. This single-channel recorder was a Minneapolis-Honeywell Model 153 x 16, potentiometer type with a 1/4 second full scale pen speed, and a range of 0-10 millivolts.

DIGITAL DATA RECORDING SYSTEM. Since data analysis on a digital computer was necessary, these records on strip chart paper had to be transferred to an input system compatible with the computer. This was done by transferring the temperature, depth, and associated time readings onto punched cards. However, this job is extremely tedious and laborious, which was one of the main reasons for the acquisition of a digital data recording system that could provide the output directly in punched tape form having a format compatible with the input system of the CDC 1604 computer. Before describing this new system, it should be noted that the system involving the strip chart recorders is still used, but primarily as a visual monitor. Thus, it is used for determining the desired running depth based on the temperature structure.

The block diagram of the digital data recording system is shown in fig. 8. The bridges were built at this Laboratory; the associated digital system was built according to our specifications by Electro-Instruments, Inc.* Most of the components are well tested, off-the-shelf items which, in general, were only slightly modified to fit into the over-all system. In general, the system sequentially records (on punched tape and/or printed tape) the temperature of 26 thermistor sensors, depth, and associated time of day. Listed below are some of the main features, functions, and limitations of the components:

1. Temperature sensors -- 26 coated or thermally lagged thermistor units.
-

*Electro-Instruments, Inc., 8611 Balboa Ave., San Diego 11, Calif.

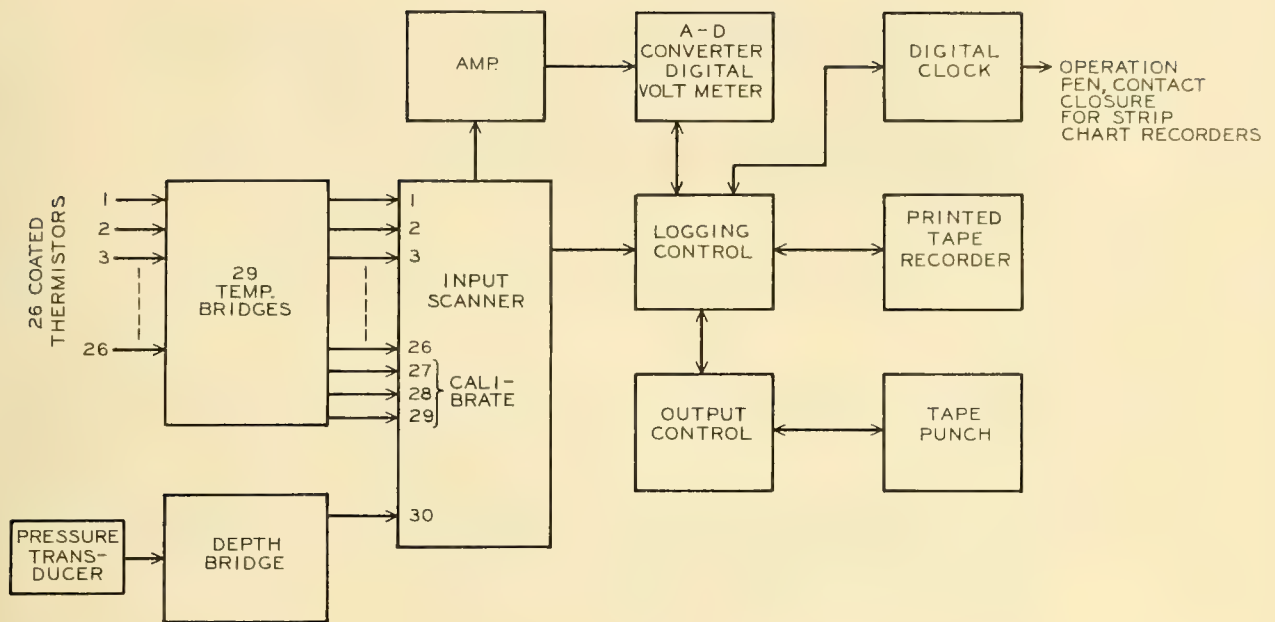


Figure 8. Digital data recording system

2. Temperature bridges -- 29 parallel bridges with one common power supply; 26 of these are for the temperature sensors and the remaining 3 provide special calibrate signals (to be described later).
3. Pressure transducer and depth bridge -- Described earlier.
4. Input scanner (Model 760-60) -- Samples 30 two-wire inputs; crossbar type with maximum switching speed of approximately 50 channels per second.
5. Amplifier (Model A-15X1) -- Single-ended input dc amplifier, with fixed gains of 1, 2, 5, 10, 20, 50, 100, 200, 500, 1000, plus adjustable gain.
6. Analog-to-digital converter (Digital Voltmeter Model 7409) -- This is the heart of the system and has three voltage ranges: 0-10, 0-100, 0-1000. When on the 0-10 volt range, it measures 0.001 volt to 9.999 volts. Maximum conversion rate is 1000 readings per second. Output to next stage is binary; the visual display is decimal.

7. Logging control (Model 806) -- Selects mode of operation, allowing the scan cycle to be started at the following intervals: 1 sec, 3 sec, 5 sec, 10 sec, 1 min, 10 min, 30 min, plus a continuous cycling rate. Also controls the presentation of the data as follows: printed tape, punched tape, both printed and punched tape.
8. Output control (Model 805) -- Provides control of the tape punch.
9. Digital clock (Model 956) -- Provides basic time base for logging control, as well as a visual display of hours, minutes, and seconds. Also enters this information on both printed and punched tapes before and after every cycle (where a cycle is defined as 26 temperature readings, 3 calibrate readings, and 1 depth reading). Clock uses a 10 kc/s temperature-compensated crystal controlled oscillator and has a stability of 1 part in 10^5 per week. In order to actuate strip-chart recorder operation pens, this clock also provides a contact closure that closes at the beginning of each minute and stays closed for 20 seconds.
10. Printed tape recorder (Model 9053) -- An electromechanical printer, which prints in decimal form at speeds up to 5 lines per second. Figure 9 shows a typical sample of this tape for one cycle, including temperatures, pressure readings, times, channel identification numbers, and polarity of voltmeter reading.
11. Tape punch, BRPE-2 type punch -- Punches temperature readings, pressure reading, and times for each cycle. However no channel identification or voltmeter polarity indications are punched. This information is provided in a 7-level code compatible with a CDC-1604 digital computer. The format also includes a carriage return before every reading, so that data from the punched tapes could easily be spot-checked by running the tapes through a Flexowriter.

CHANNEL NUMBERS		READING POLARITY	READINGS				
↓	↓	↓	↓	↓	↓	↓	
	1 5		1 6	2 6			TIME AT END OF SCAN CYCLE (HOURS, MINUTES, SECONDS)
2 9	+		1 8	0 3			← DEPTH READING
2 8	+		8 9	9 9			← 18°C CALIBRATE
2 7	+		4 0	9 0			← EXTERNAL CALIBRATE
2 6	+		0 9	9 9			← 10°C CALIBRATE
2 5	+		4 0	8 7			
2 4	+		4 0	6 0			
2 3	+		4 0	9 1			
2 2	+		4 0	9 0			
2 1	+		3 9	4 6			
2 0	+		4 0	1 6			
1 9	+		4 0	1 3			
1 8	+		4 1	1 1			
1 7	+		4 1	1 4			
1 6	+		4 0	4 6			
1 5	+		3 9	8 9			
1 4	+		3 9	5 4			
1 3	+		4 0	3 5			
1 2	+		3 9	8 5			
1 1	+		4 1	0 3			
1 0	+		4 0	3 2			
0 9	+		3 9	9 9			
0 8	+		3 9	6 6			
0 7	+		3 9	9 0			
0 6	+		4 1	1 4			
0 5	+		4 1	1 0			
0 4	+		4 1	0 2			
0 3	+		4 0	1 3			
0 2	+		3 9	7 2			
0 1	+		4 1	0 9			
0 0	+		4 0	7 1			
1 5			1 6	1 8			← TIME AT BEGINNING OF SCAN CYCLE (HOURS, MINUTES, SECONDS)

26 TEMP
READINGS

(eg 12.966 °C)

Figure 9. Typical (simulated) sample of one scan cycle from printed tape recorder

The following describes a typical operation, including adjustments and calibrations, of the digital data recording system. The temperature bridges were balanced to give zero voltage output at 9°C, and thus the digital voltmeter read 0.000 volt at 9°C. The bridge span was adjusted to give approximately 20 millivolt output at 19°C, and the amplification then adjusted (approximately 500) so as to produce 9.999 volts on the digital voltmeter. Therefore, the voltmeter readings had a sensitivity of 1 volt per 1°C, and the least count of the voltmeter corresponded to 0.001°C. These adjustments are made by switching calibrate resistors corresponding to even degree temperatures into bridge number 28 (fig. 5B). After calibration is completed, this channel monitors a resistance located on the strut. Bridges 27 and 29 also provided calibration voltages once every scanning cycle. The resistor providing the bridge number 27 calibration value corresponded to 10°C while that of bridge number 29 corresponded to 18°C. The computer can then, while reading the tapes, determine from these two calibrate voltages the correct sensitivity to apply to readings in that particular scan cycle. In this manner any changes as a function of time in the power supply batteries, gain of the amplifier, etc., can be compensated for continuously.

The pressure bridge output (fed to channel number 30 of the input scanner) is amplified at the gain determined by the temperature circuits. The result is a maximum depth capability (digital voltmeter reading 9.999 volts) of approximately 400 feet, with a readout least count of 0.04 feet.

The logging cycle rate is normally set at 10 seconds. Thus, every 10 seconds a cycle, consisting of a time, 26 temperature readings, 3 calibrate readings, 1 depth reading, and another time reading, is indicated. This means that the sampling speed is approximately 3-4 readings per second. In this mode both the tape punch and printed tape recorder are used at all times in recording data.

Several features of this digital data recording system provide flexibility for future expanded uses. A considerable effort was spent in specifying the system so that a maximum number of operating modes could be realized with a minimum number of changes.

Thus, by cutting the printed recorder out of the system and using only the tape punch, the sampling rate may be increased to 9-10 channels per second. By using a faster tape punch (soon to be available), the sampling may be increased to approximately 17 channels per second. If the recorder is not the limiting factor, the sampling rate could be increased to 50 channels per second (limited by present input scanner). Procurement of a magnetic tape recorder would be necessary in this case for recording purposes. Finally, the digital voltmeter has a maximum possible conversion rate of 1000 readings per second. Also, the number of inputs to be scanned and the time between cycles or logging times can both be varied easily.

OVER-ALL ACCURACY AND PRECISION

As described in Appendix B, the thermistors are calibrated* in a tank in which the water can be maintained at an indicated temperature with a high degree of precision (see below). The accuracy of the temperature setting is determined by the accuracy of a platinum thermometer which has been compared with a recently calibrated secondary temperature standard. The result is shown in fig. 10, curve A, which includes the accuracy of the temperature bath as well as the bridge measurement.

The linearization of the thermistor-bridge combination as described in Appendix B is not perfect, although it is so good that corrections are not applied. The small inaccuracies that do occur are expressed in one form in the appendix. In practice (because it is easier to do so), the null and the sensitivity are calibrated at 9°C and 18°C respectively; the relation between temperature and voltage is taken as a straight line between these points. For this case, the inaccuracies are given in figure 10, curve B.

The error in accuracy of the temperature indicating system is the algebraic sum of the individual errors in accuracy; it is given in figure 10, curve C. Here the errors in accuracy of the digital data system are not included. The accuracy of the digital

*Reference 3, page 55

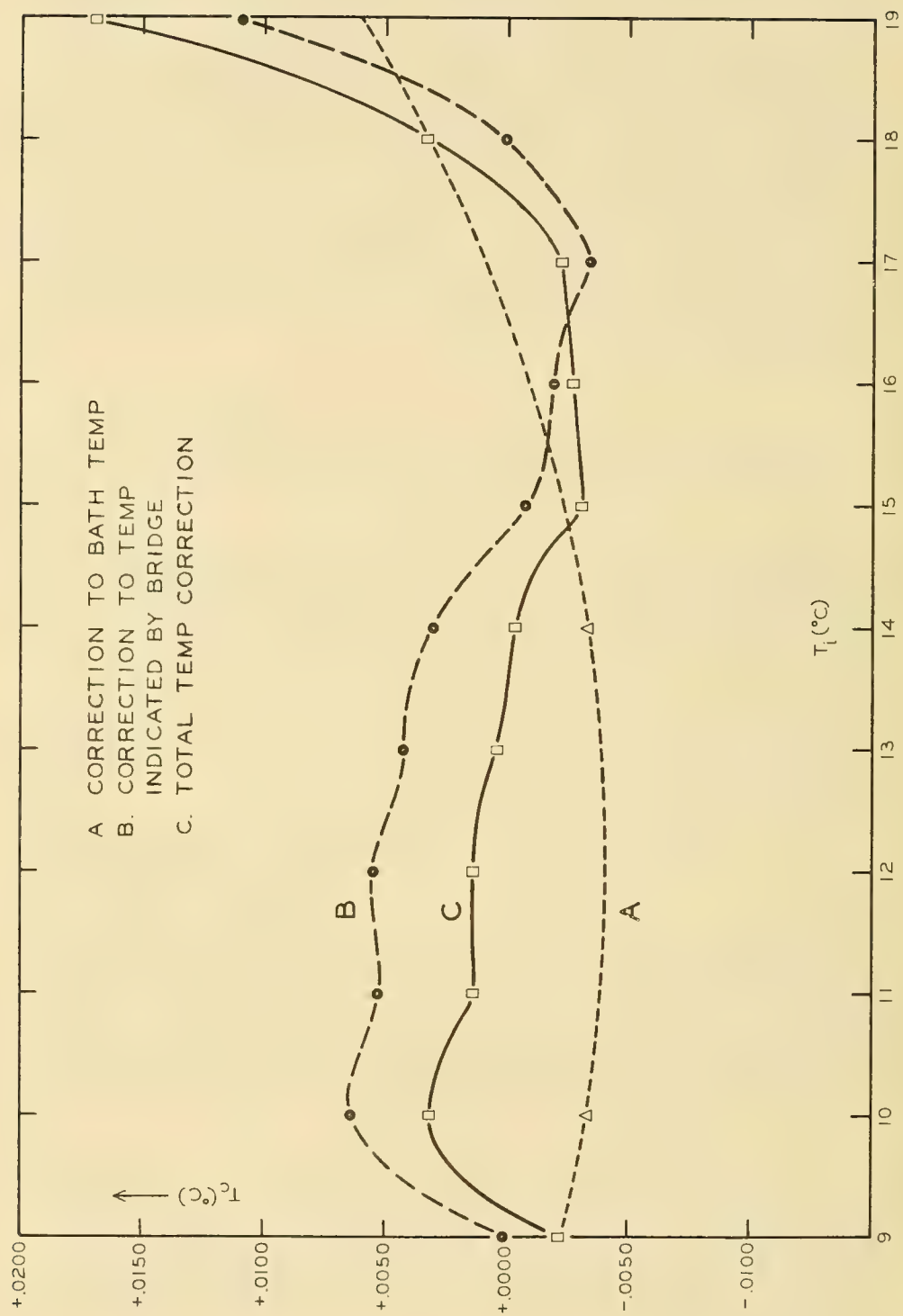


Figure 10. Temperature corrections

system is very high, of the order of ± 0.014 percent, owing to the great accuracy of the digital voltmeter and to the linearity of the amplifier. A correction curve could be obtained by using the whole series of calibrate resistors.

The precision* with which the calibrating tank can be maintained at an indicated temperature is $\pm 0.001^{\circ}\text{C}$. The precision of the measurement of the resistance of a thermistor is ± 0.09 ohm (equivalent to 0.0007°C); lack of precision is due in part to failure to maintain temperature constant.

In addition, the value of the resistance of the equivalent normalized resistor at each of the calibration temperatures is taken as the mean of the resistance of the individual thermistors in the set. The probable error of these readings is taken as a quasi-precision of this resistance, and was found to be equivalent to $\pm 0.002^{\circ}\text{C}$. Note that this includes the two preceding precisions.

The precision with which a given resistance can be determined (this includes fluctuations in the bridge and in the digital data system) is obtained by repeated readings of a given calibrate resistor; this gives a probable error for the measurement of the resistance that corresponds to $\pm 0.0025^{\circ}\text{C}$.

The over-all precision of the temperature measurements is the square root of the sum of the squares of the above two individual sources of probable error. The over-all precision is $\pm 0.0032^{\circ}\text{C}$.

The errors in the temperature sensors introduced during the field trip are difficult to assess. Calibrations are made before and after the trip; the differences in calibration are usually within the probable error of the measurements. The effect of depth (i.e., pressure) on the calibration is difficult to determine, but tests elsewhere indicate that the effect is negligible at depths up to 500 ft.**

*Precision here will be taken to mean the probable error in the value of a reading repeated a number of times.

**Reference 4, page 55

The effect of temperature variation in the environment on the temperature sensing system is not known. The temperature of the usual environment is quite constant, and thus this difficulty is not important. In any event, the use of the automatic calibration removes these sources of inaccuracy.

The pressure transducer which indicates depth has errors due to nonlinearity and hysteresis specified to be 3/4 percent of full scale reading (1000 ft). In practice, only a small interval is used during a test run, and hence nonlinearity and hysteresis are greatly reduced. The depth gage in the submarine control room is used to calibrate depth above and below the thermocline (say an interval of 50 ft) and at one or more points in the thermocline. Not enough work has been done to give the precision or accuracy of these measurements; the error in the relative depth is probably a fraction of a foot.

DATA REDUCTION

It is clear from the above descriptions that the record obtained gives the indicated temperature at several points on the strut, as well as the strut depth. The object of the data reduction described here is to yield the depths of one or more true isotherms as a function of position along the track of the recording submarine. The analysis of such a signal (see fig. 13 for an example) is not considered here.

LINEARIZATION OF THE OUTPUT OF THE THERMISTOR BRIDGES

When the techniques of Appendix B are properly applied, the output voltage is so very nearly a linear function of the temperature (over a restricted range) that no correction need be applied. If the bridge parameters are not quite optimum or if too large a temperature range must be spanned, there is a departure from linearity.

The indicated temperature, T_i , can be made linear by adding a temperature correction* T_c , to give the true (linear) temperature, T . Thus

$$T = T_c + T_i \quad (1)$$

Typical values of** T_{cj} vs. T_{ij} are shown in table 1. (Here the second subscript indicates the j th point of observation and $j = 0, 1, \dots, n$.) The approximation of T_c by a harmonic series, rather than a polynomial, is indicated. The use of the expression "harmonic" is not quite proper if the intervals are not equal. (It is difficult to measure the T_{cj} 's for equal intervals of T_{ij} .) Also, it is usual to consider both sine as well as cosine series. If this is done, the highest harmonic, k , in the series must satisfy the condition $2k + 1 \leq n$. It is clear from the table that higher harmonics are present. For this reason, the function $T_c(T_i)$ is continued to the left of the initial value as an even function of that point. Then harmonics up to and including the n th can be used; the series is the cosine series:

$$T_c = \sum_{j=0}^n a_j \cos \left[j\pi \frac{(T_i - T_{i0})}{T_{in} - T_{i0}} \right] \quad (2)$$

Now the a_j are chosen to minimize the residual equation

$$\sum_{k=0}^n R^2 = \sum_{k=0}^n \left\{ T_{ck} - \sum_{j=0}^n a_j \cos \left[j\pi \frac{(T_{ik} - T_{i0})}{T_{in} - T_{i0}} \right] \right\}^2 \quad (3)$$

where T_{ik} is the k th indicated temperature. The partial derivatives with respect to a_j are set equal to zero. When done for each a_j , the result is a set of $(n + 1)$ normal equations:

*Note that T_c should be given as a function of T_i .

**These values were obtained before the techniques of Appendix B were developed.

$$\sum_{j=0}^n \left\{ a_j \sum_{k=0}^n \cos \left[j\pi \frac{(T_{ik} - T_{i0})}{T_{in} - T_{i0}} \right] \cos \left[r\pi \frac{(T_{ik} - T_{i0})}{T_{in} - T_{i0}} \right] \right\} \\ = \sum_{k=0}^n T_{ck} \cos \left[r\pi \frac{(T_{ik} - T_{i0})}{T_{in} - T_{i0}} \right] \quad (4)$$

for $r = 0, 1, \dots, n$

In matrix notation, this becomes

$$\alpha_r \cdot \alpha_j = \beta_r \quad (5)$$

As an example, consider table 1. In the first row is given the indicated temperature. In the second row, the measured temperature correction is given. The introduction of these quantities into equation (4) gives the set of a_j 's in the third row. Putting these a_j 's into equation (2), gives a set of T_c 's for the specified T_{ik} , shown in the fourth row. Comparison of the fourth row with the second row shows that the fit of the harmonic series to the measured values of T_c is exact except for small round-off errors. This is to be expected since the number of parameters determined is the same as the number of points fitted.

TABLE 1. VALUES OF T_{cj} VS. T_{ik} .

j	0	1	2	3	4	5
T_{ik}	8.62	10.93	13.05	14.97	17.20	19.27
T_{ck}	+ .32	+ .28	+ .10	-.07	-.22	-.31
a_j	.02448964	.31270375	-.0035209249	.0020677841	-.015928043	.0002284838
T_c	.32000001	.28000003	.10000001	-.07000018	-.22000004	-.31000001

DEPTH COMPENSATION

To get the depth of the thermistor beads (in the next section it is shown how the depth of the isotherms is subsequently obtained) a pressure transducer is used. The zero adjust and the sensitivity are set in the laboratory and/or at dockside, so that the depth indicated by the pressure transducer system is the same as the keel depth as indicated by the master depth gauge (with the submarine at zero bubble). Underway, checks are made at various depths. Since the pressure transducer is quite linear, a linear interpolation between two check points that straddle the thermocline are used. Thus, the keel depth at zero bubble is given by

$$Z = az + b, \quad (6)$$

where Z is the true keel depth, and z is that indicated by the pressure transducer. At the two check points

$$Z_j = az_j + b, \quad j = 1, 2 \quad (7)$$

and thus the constants a and b can be determined. Now a and b may drift slightly with time. If necessary, the values of a and b may be linearly interpolated between check points.

Finally, the depth z_k , of the k th bead is given by

$$Z_k = az + (b - b_k) \quad (8)$$

where b_k is the vertical distance of the thermistor to the keel.

INTERPOLATION FOR DEPTH OF A GIVEN ISOTHERM

The data obtained in the field for a given sequence of readings are the temperatures at several points on the strut and the strut depth. These data are not immediately useful. They can be used, however, to give isotherms along the track of the recording platform.

The temperature at any point along the strut was originally approximated by a harmonic series, just as was T_c in the analysis above. The resultant curve exhibited marked maxima and minima and hence was undesirable. A power series approximation was tried with even worse results. The difficulty arises from the fact that, although the temperature rises monotonically as one goes up the strut, part of the strut may be in nearly isothermal water.

This leads to values of $\frac{\partial Z}{\partial T}$ that are very high. This high slope is reduced by first approximating the temperature dependence on Z by the best straight line, i. e., by the line of regression. Thus,

$$Z_o - \frac{\sum_{k=1}^n Z_k}{n} = b_{Z \cdot T} \left(T_o - \frac{\sum_{k=1}^n T_k}{n} \right) \quad (9)$$

gives the points (Z_o, T) on the line of regression, where

$$b_{Z \cdot T} = \frac{\frac{n}{\sum_{k=1}^n T_k Z_k} - \frac{\sum_{k=1}^n T_k}{\sum_{k=1}^n T_k} \frac{\sum_{k=1}^n Z_k}{n}}{\frac{n}{\sum_{k=1}^n T_k^2} - \left(\frac{\sum_{k=1}^n T_k}{n} \right)^2} \quad (10)$$

Now form a new coordinate system with new origin at

$$\left(\frac{\sum_{k=1}^n T_k}{n}, \frac{\sum_{k=1}^n Z_k}{n} \right)$$

and with the axes rotated by $\theta = \tan^{-1} b_{Z \cdot T}$. (Thus the angle of the tangent to the curve is reduced by θ .) The variables (T, Z) of the new coordinate system in terms of the old coordinates are

$$T' = (Z - \bar{Z}) \sqrt{\frac{b_{Z \cdot T}}{b_{Z \cdot T}^2 + 1}} + (T - \bar{T}) \sqrt{\frac{1}{b_{Z \cdot T}^2 + 1}} \quad (11A)$$

$$Z' = (Z - \bar{Z}) \frac{1}{\sqrt{b \frac{Z^2}{Z \cdot T} + 1}} - (T - \bar{T}) \frac{b \frac{Z \cdot T}{Z \cdot T}}{\sqrt{b \frac{Z^2}{Z \cdot T} + 1}} \quad (11B)$$

where

$$\bar{T} = \frac{\sum_{k=1}^n T_k}{n}, \quad \bar{Z} = \frac{\sum_{k=1}^n Z_k}{n}$$

Now approximate* Z' (T') by

$$Z' = a_{n-1} (T')^{n-1} + a_{n-2} (T')^{n-2} + \dots + a_1 (T') + a_0 \quad (12)$$

Now, the several values of data points (T_k, Z_k) are substituted in equation (11) and for each (T'_k, Z'_k) , there will be a particular equation (12). There will be n such simultaneous equations in the n unknowns:

$$a_{n-1}, a_{n-2}, \dots, a_1, a_0$$

Once these coefficients are determined, they can be put into equation (12). This equation gives the interpolation for T' values of Z' .

Now we want to assign specific values to T and solve for the corresponding values of Z given by the above interpolation scheme. A direct substitution of equation (11) into equation (12) would result in a polynomial in Z . This would be difficult to handle, so the following iterative scheme is used (fig. 11). Select $T = T_S$, one of the arbitrarily chosen isotherm values. Then, first, intersect

*The harmonic series would probably give a better fit, but the transformation to the original coordinate system is rather intractable.

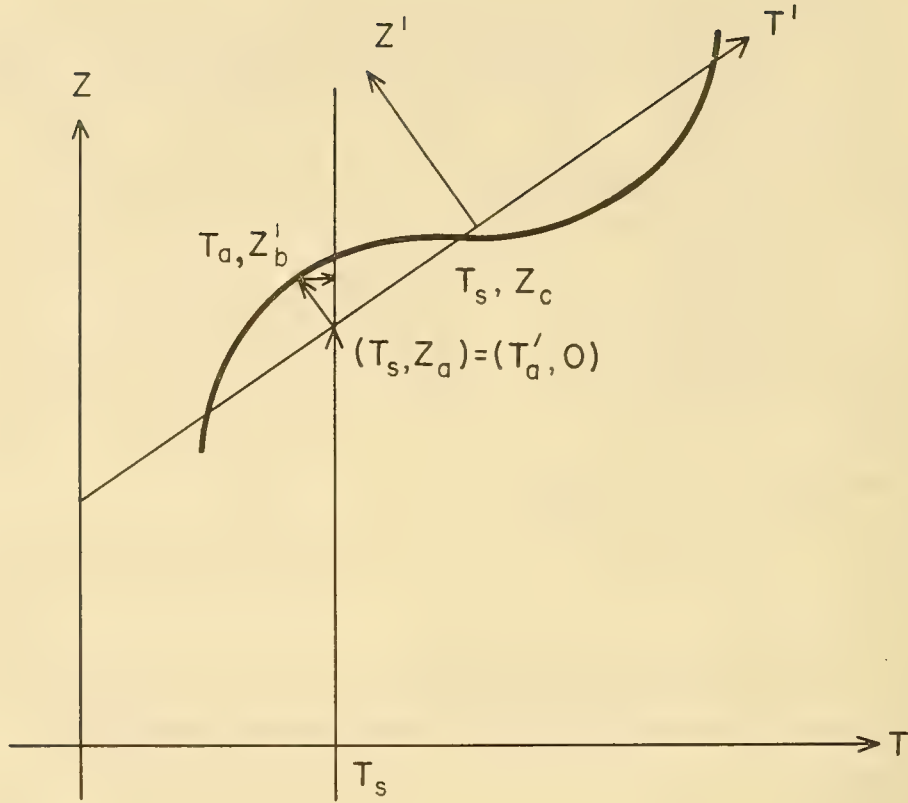


Figure 11. An iterative method for determining a Z associated with a given T

$T = T_s$ with line of regression. This gives Z_a . From equation (9), we have

$$Z_a = \bar{Z} + b_{Z \cdot T} (T_s - \bar{T})$$

Second, this point has the coordinates (T_s, Z_a) . In the new coordinate system, it has coordinates $(T'_a, 0)$, where T'_a can be computed by substituting T_s and Z_a into equation 11a. Pass the line $T' = T'_a$ until it intersects the interpolation curve at b. This point has coordinates $(T'_a = T'_b, Z'_b)$, where Z'_b is computed from equation (12) by putting T'_a for T' . Next, through (T'_b, Z'_b) , pass a line parallel to T axis until it intersects line $T = T_s$ at the

point (T_s, Z_c) where $Z_b = Z_c$ can be computed by substituting T'_a and Z'_b into the expression

$$Z = \frac{b_{Z.T}}{\sqrt{b_{Z.T}^2 + 1}} T' + \frac{1}{\sqrt{b_{Z.T}^2 + 1}} Z' + \bar{Z}$$

Finally, repeat until Z is close enough to the limit.

An example of an attempt to fit the temperature data along the strut is given in figure 12. Interpolations of the temperature at half-degree intervals are given by the harmonic method and by the polynomial method in a rotated coordinate system. The harmonic series obviously gives a very bad fit; a polynomial (not shown) gives an even worse fit. The polynomial in the rotated coordinate system gives a fairly good fit. It would have been more instructive, though, to have calculated the depth at the temperature data points.

FINAL FORM OF THE REDUCED DATA

For figure 12, the interpolation scheme described above was used to determine the position of a number of isotherms relative to the strut. It is shown earlier how this result can be used to give the depth in the sea for the various isotherms. These processes are repeated for each sequence of temperature and pressure readings. The result may be plotted as a function of time or position. In figure 13, the depth of a number of isotherms is given as a function of time for the case of an array being carried at a uniform horizontal speed and at such a depth that the thermocline is straddled.

Because the strut does not straddle the thermocline uniformly, the procedure in the section on interpolation does not give depths for all isotherms. A technique, not to be discussed here, has been devised to fill in isotherm gaps; these are indicated as dotted curves. Further analysis (i. e., auto-correlations, power spectra, filtering, etc.) of the reduced data is likewise not discussed here. It should be noted, however, that the frequencies of

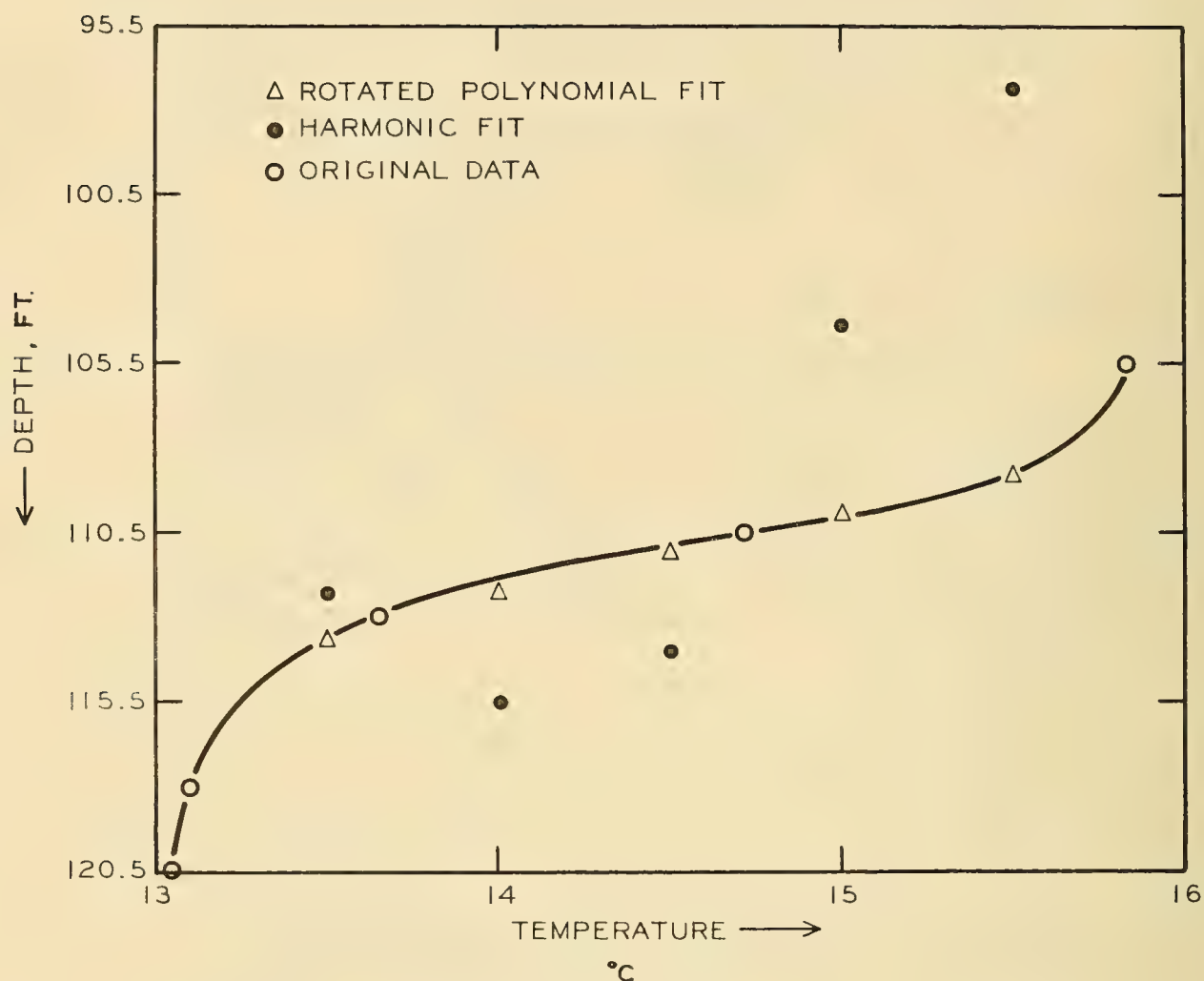


Figure 12. Fit of temperature data along strut

signals in the record are not, in general, the frequencies of the internal waves in the earth's reference system; they are what naval architects call "frequency of encounter". There is a Doppler shift, depending on the relative bearing of the propagation vector. In fact, at certain narrow intervals of bearing, there is not even a one-to-one correspondence between the two kinds of frequencies throughout the spectrum.

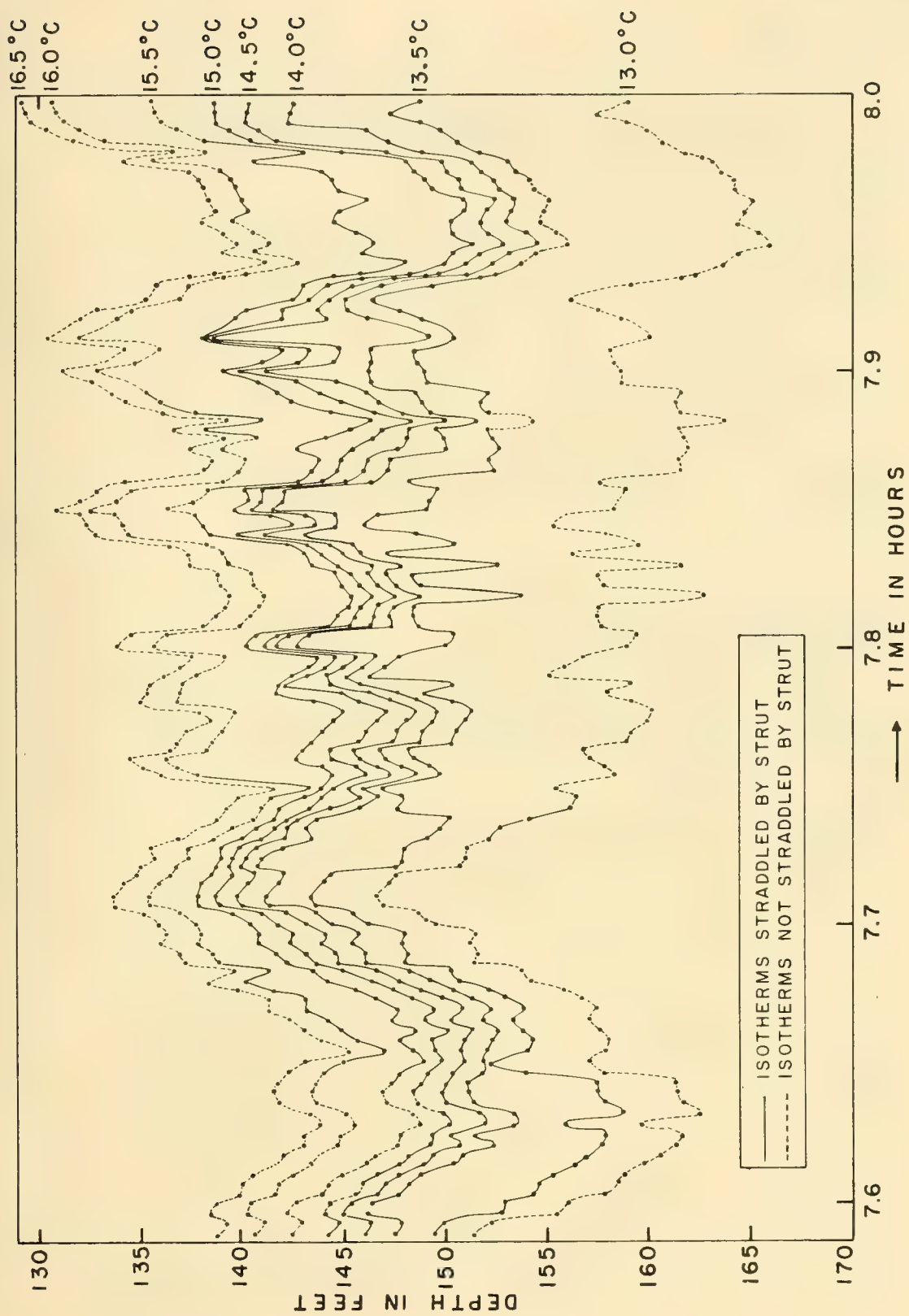


Figure 13. Depth of isotherms as a function of time obtained from an array moving at the thermocline at a fixed horizontal speed

RESULTS

1. A submarine-mounted vertical array of thermistors for temperature measurements at several levels was designed, constructed, and tested.

2. The precision (repeatability), using probable error as criterion, is $\pm 0.0032^{\circ}\text{C}$. The accuracy (absolute) is $\pm 0.003^{\circ}\text{C}$ over the usable range.

3. Data output is in digital form.

4. The digital data are reduced to give depth of isotherms (i.e., internal waves) as a function of position along the track. Techniques for accomplishing this with a high speed computer have been devised and tested.

RECOMMENDATIONS

Accord recognition to the importance of the submarine as a working platform for the oceanographer. Use a submarine, especially during the season of well-developed thermoclines, for the study of internal waves.

APPENDIX A: MODIFICATION OF L. J. ANDERSON'S COMPENSATION METHOD FOR THERMISTOR BEADS

When a number of temperature sensors are used in an array, it is desirable for all the sensing elements to have the same variation in electrical parameter with temperature. In the instrument described here, the temperature sensor is the thermistor. The thermistor is a semiconductor device with a large negative temperature coefficient of resistance. The resistance, R , of a thermistor as a function of temperature is given empirically by the expression

$$R = a e^{b/T} \quad (\text{A1})$$

where a and b are constants and T is the absolute temperature. The constants, a and b , vary from thermistor to thermistor, and the fit of the empirical expression to the thermistor characteristics may vary. This appendix discusses the means by which the resistance vs. temperature characteristics of the several thermistors may be compensated to produce one common characteristic.

L. J. Anderson* developed one technique for thermistor beads. In his method, the resistance of each bead of a group is determined precisely at two temperatures, T_1 and T_2 . Let the corresponding resistances for a typical bead be r_1 and r_2 . Two resistance values, R_1 and R_2 , are chosen. R_1 is chosen equal to the greatest thermistor resistance, r_1 , in the group, and R_2 is chosen equal to the lowest thermistor resistance, r_2 (fig. A1). For each thermistor, a resistance, b , is placed in series, and a resistor, a , is placed in parallel. The values are so chosen that the net circuit resistance be R_1 at T_1 , and R_2 at T_2 . The formulas given by Anderson for these values are:

$$b = \frac{1}{2} \left[(R_2 + R_1) - (R_2 - R_1) \left(1 + \frac{4 r_1 r_2}{(R_2 - R_1) (r_2 - r_1)} \right)^{\frac{1}{2}} \right] \quad (\text{A2})$$

*Reference A1, page 55

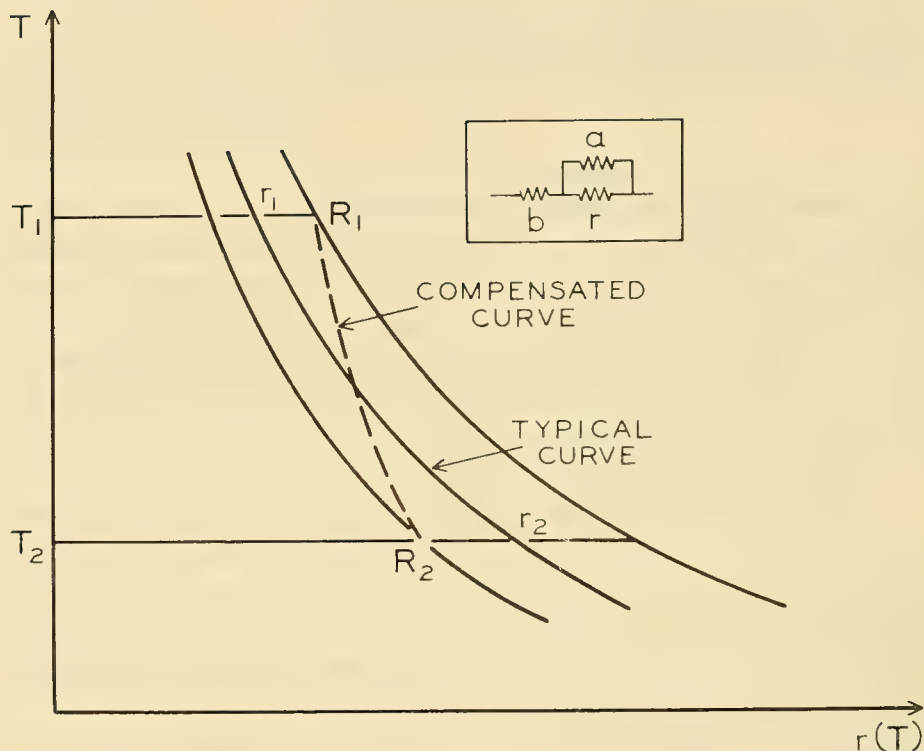


Figure A1. Anderson's compensation method

$$a = \frac{r_2 (R_2 - b)}{r_2 + b - R_2} \quad (\text{A3})$$

Now the effect of the resistance b is to translate the curve $r(T)$ to the right, while the effect of resistance a is to rotate $r(T)$ clockwise and to displace it a little to the left although a slight distortion also occurs (figure A2). Note in figure A1 that the amount of rotation required to fit Anderson's compensated thermistor curve is considerable compared to the amount of rotation in figure A2 required to make the most divergent $r(T)$ curves parallel. In other words, a simple translation alone can nearly cause coincidence. Only a slight rotation is then necessary. Two advantages result: (a) there is a minimum distortion of the curves, and (b) the (negative) rate of change of resistance with temperature (a measure of sensitivity) is greater.

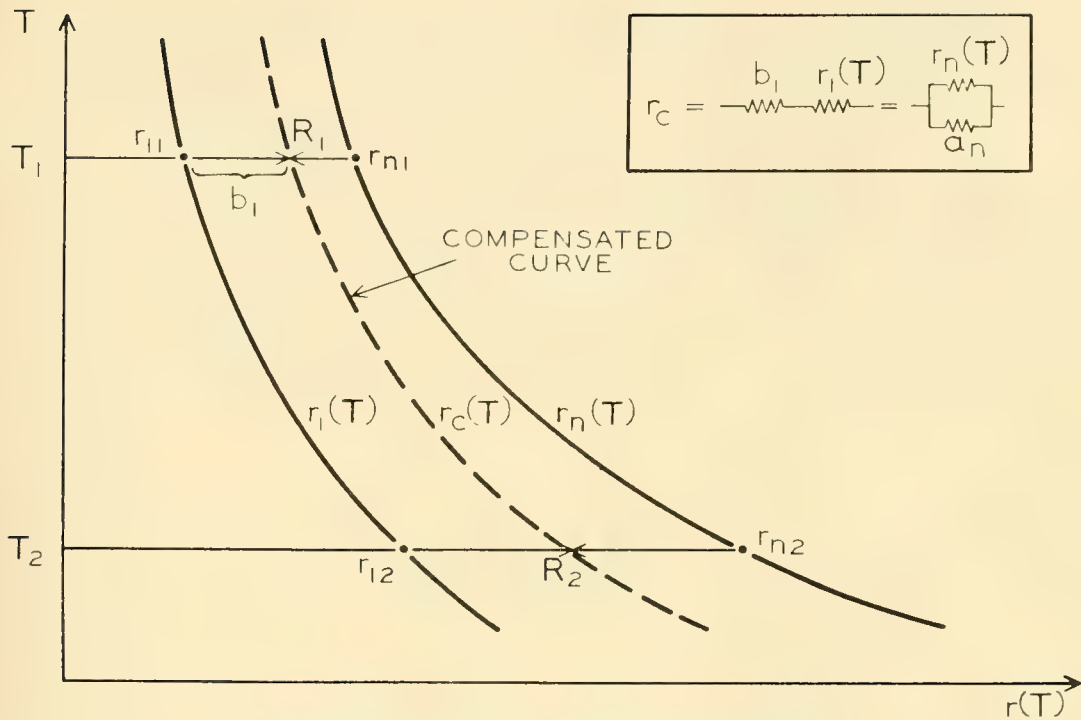


Figure A2. New compensation method

The procedure recommended here is as follows. Plot the calibration curve $r(T)$ for a group of thermistor beads. Most of them will be nearly parallel; reject those which are not. Put only a resistor, b_1 , in series with the thermistor bead for which the calibration curve (curve 1) is the steepest, and put only a resistor, a_n , in parallel with the thermistor bead whose calibration curve (curve n) lies farthest to the right. Choose b_1 and a_n such that the net resistance of the two circuits are equal at T_1 and T_2 , i.e.,

$$b_1 + r_{11} = \frac{a_n r_{n1}}{a_n + r_{n1}} \quad (\text{A4})$$

$$b_1 + r_{12} = \frac{a_n r_{n2}}{a_n + r_{n2}} \quad (\text{A5})$$

where:

$$r_{11} = r_1(T_1), r_{12} = r_1(T_2), r_{n1} = r_n(T_1) \text{ and } r_{n2} = r_n(T_2)$$

These can be solved simultaneously to give:

$$b_1 = \frac{1}{2} \left\{ - (r_{11} + r_{12}) + \left[(r_{11} + r_{12})^2 + \frac{4 r_{n1} r_{n2} (r_{12} - r_{11})}{r_{n2} - r_{n1}} - 4 r_{11} r_{12} \right]^{\frac{1}{2}} \right\} \quad (\text{A6})$$

$$a_n = \frac{r_{n2} (b_1 + r_{12})}{r_{n2} - b_1 - r_{12}} \quad (\text{A7})$$

For the remaining beads, formulae (A2) and (A3) may be used provided that one takes:

$$R_1 = r_{11} + b_1, \text{ and } R_2 = r_{12} + b_1 \quad (\text{A8})$$

Note that it may be possible to get a negative value for b in equation (A2). This would happen if the $r(T)$ curve lies quite close to the one farthest to the right, and is appreciably less steep. It must be rotated less to get it parallel to curve 1. As pointed out before, the rotation is accompanied by a translation to the left. If the rotated curve falls to the right of the rotated curve n (or what is equivalent to curve 1, translated b_1 to the right) it cannot be translated to the left except by a negative series resistor. In such a case, this curve must be used in place of the one farthest to the right in computing b_1 and a_n .

Note that the only rotation of a curve possible is clockwise. This reduces sensitivity. The curve chosen for zero rotation is the steepest curve; the other curves can be rotated into parallelism with it, but it cannot be rotated into parallelism with any of the others. The value of b_1 obtained here is minimum. Values larger than this would give equally well compensated curves. The proper

value of b_1 is determined by other considerations, such as the degree of linearization when the compensated thermistors are put into a bridge circuit.

The following is an example of the new compensation method. Eight Veco thermistors (No. 32A1) were used (see fig. A3). Good compensation was desired from 9°C to 19°C. T_1 and T_2 were chosen to be 17°C and 13°C respectively. $r_{11} = 2564$ ohms, $r_{12} = 3004$ ohms, $r_{n1} = 3006$ ohms, $r_{n2} = 3522$ ohms.

From eqs. (A6) and (A7):

$$b_1 = 228.65 \text{ ohms}$$

$$a_n = 39,348 \text{ ohms}$$

From eq. (A8):

$$R_1 = 2792.65 \text{ ohms}$$

$$R_2 = 3232.65 \text{ ohms}$$

Using eqs. (A2) and (A3), the results in table A1 are obtained.

TABLE A1. VALUES OF a_i AND b_i .

Thermistor No.	a_i (ohms)	b_i (ohms)
1	101,540	154.7
2	∞	229.5
3	39,252	0.7
4	60,815	115.1
5	158,934	183.3
6	73,216	110.7
7	40,213	9.5
8	58,801	86.5

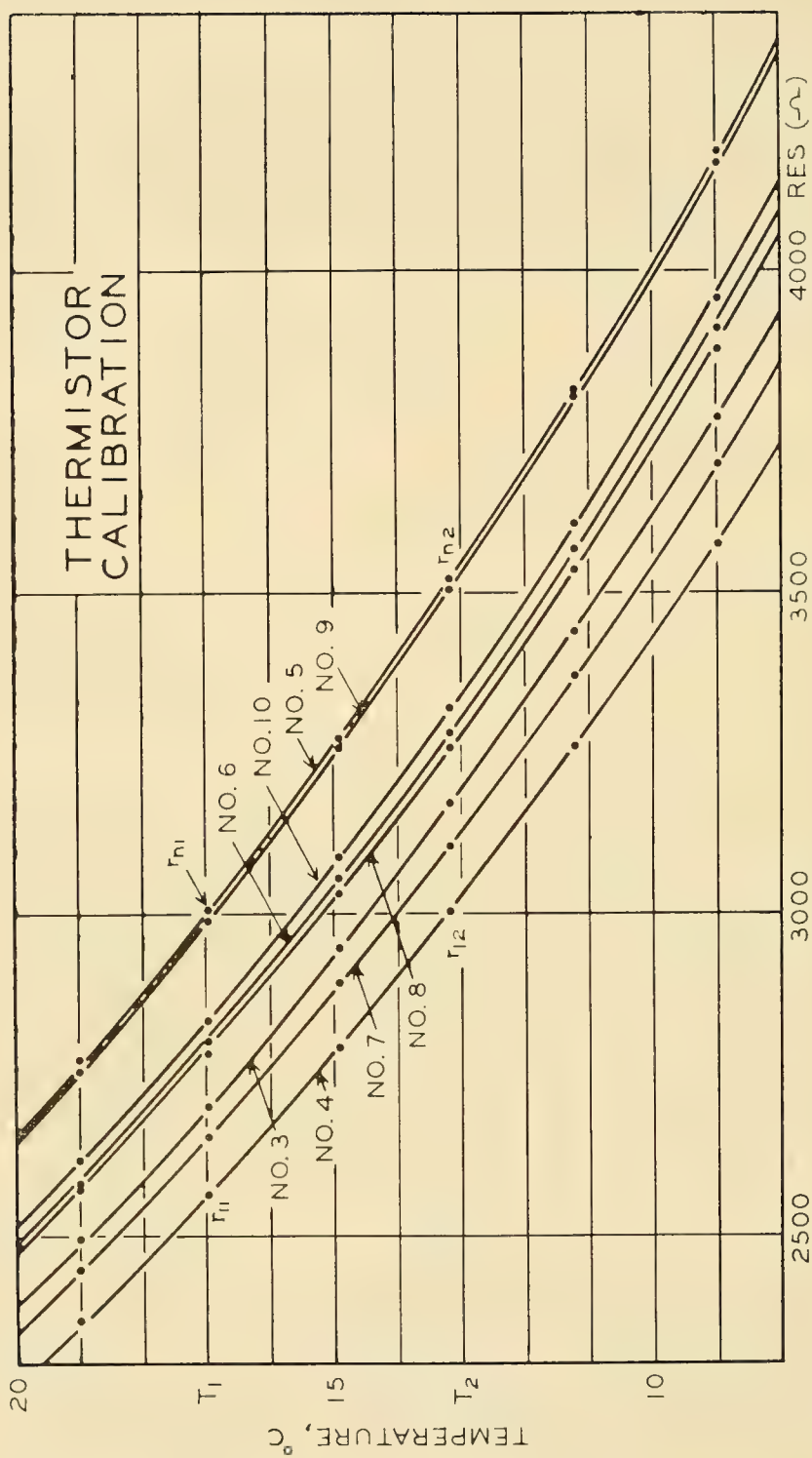


Figure A3. Resistance vs. temperature curves for the eight Veco thermistors used

Let R_{0i} be the equivalent resistance of the combination a_i , b_i , and the i th thermistor resistance. The calculated values of R_{0i} are shown in table A2.

TABLE A2. CALCULATED VALUES OF R_{0i} .

Temperature (°C)	Thermistor No.								Mean R_{0i}
	1	2	3	4	5	6	7	8	
9	3791.6	3801.5	3784.1	3789.8	3799.1	3792.7	3786.9	3791.4	3792.1
11	3482.9	3488.5	3481.9	3483.6	3487.2	3484.7	3484.8	3484.1	3484.7
13	3232.65 —————→								
15	3019.5	3022.5	3022.6	3023.1	3021.7	3022.1	3021.6	3021.3	3021.8
17	2792.65 —————→								
19	2593.7	2597.5	2593.3	2594.7	2594.2	2595.4	2591.3	2592.9	2594.1

Although the compensated resistance for the specified temperatures have been obtained by calculation, these calculations are made using measured values of resistance for the individual thermistors.

Deviations from the mean R_{0i} values were largest at 9°C. At this temperature, the greatest deviation (thermistor No. 2) was only 0.25 per cent from the mean. If desired, deviations at this end of the range could be made smaller by choosing T_2 closer to 9°C.

APPENDIX B: OPTIMUM PARAMETERS FOR LINEARIZING THERMISTOR TEMPERATURE BRIDGES

INTRODUCTION

One of the simplest ways of utilizing a thermistor for the measurement of the temperature is to measure the voltage drop across it due to the passage of current through it.* If the current is constant, the voltage drop is nonlinear as can be seen from the empirical expression $R = a \exp(b/T)$ for thermistor resistance, where a and b are constants, and T is absolute temperature.** However, by using a constant potential source to drive current through a resistor of proper value in series with the thermistor, a certain degree of linearity in the variation of the voltage drop across the thermistor with change in temperature is achieved. The purpose of this appendix is to show how to choose the optimum value of series resistance, and the resultant degree of linearity which can be achieved with typical thermistors.

The following is a short review of some of the work that has been done in this field.

Burke, during temperature studies at NEL worked with Wheatstone bridges and found an expression to determine the optimum value of resistance, r (fig. B1). The condition for linearity was such that two equal increments of voltage output corresponded to two equal temperature increments. This answer was found in terms of the thermistor resistance at three discrete equally spaced temperatures (R_a , R_b , and R_c ***; the thermistor resistance at the low, middle, and high end respectively of the desired temperature range) (fig. B2) and is shown below:

* The maximum current through the thermistor should be small enough so as to produce negligible elevation in temperature relative to that of the fluid in which it is immersed.

** This is equation (A1) of Appendix A.

***Symbols have been changed from some of the original papers quoted, for consistency.

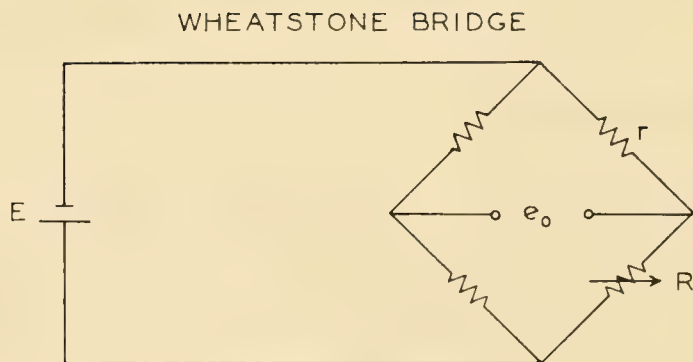


Figure B1. Wheatstone bridge

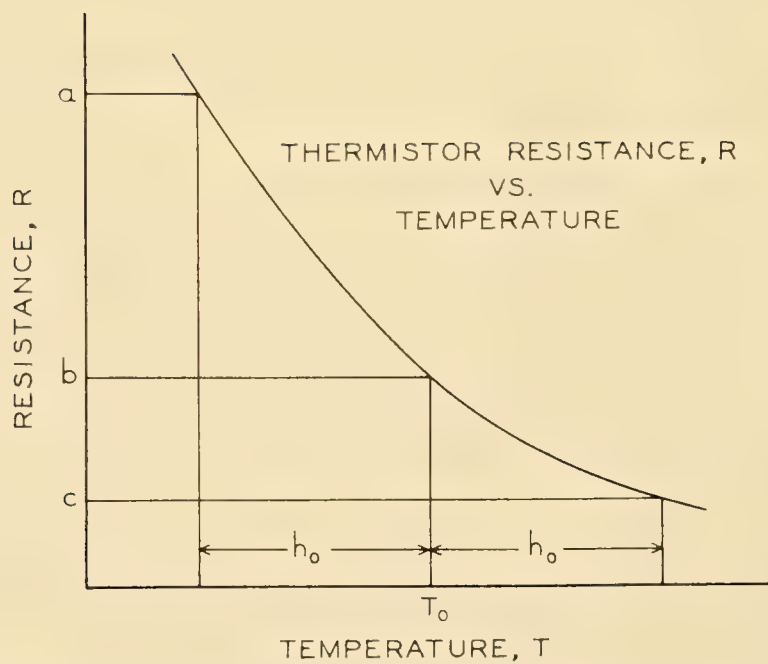


Figure B2. Thermistor resistance vs. temperature

$$r = \frac{R_b(R_a + R_c) - 2R_a R_c}{R_a + R_c - 2R_b} \quad (\text{B1})$$

Kaufman* described a circuit called the asymmetric half-bridge, which consists of the thermistor, R , a series resistor, r , and two batteries, E_1 and E_2 (fig. B3). For a given thermistor and series resistor, the sum of $E_1 + E_2$ adjusts the sensitivity while the ratio E_2/E_1 adjusts the balance point of the bridge. Thus for zero output at temperature T_x , with corresponding thermistor resistance R_x ,

$$\frac{E_2}{E_1} = \frac{r}{R_x} \quad (\text{B2})$$

The half-bridge offers some advantages over the full Wheatstone bridge. Half-bridges are capable of greater power output for use with noninfinite loads because of their lower output impedance. For multiple bridges in parallel, used with one common dual power supply, a common ground at the center tap of the dual supply, may be used. Kaufman** mentions the importance of this advantage in the application of telemetering output signals.

In both of the above mentioned articles by Kaufman, he discusses half-bridges for use only with essentially linear temperature elements, such as platinum resistance thermometers. For best linearity in this case, a very large value of series resistor, compared to the temperature element resistance, is needed. Then the bridge becomes essentially a constant current device and the IR drop as a function of temperature across the temperature element is nearly linear, since the element resistance itself is nearly linear with temperature.

Beakley*** worked with thermistors in describing a bridge circuit consisting of a thermistor, R , a series resistor, r , and only one battery, E (fig. B4). He used the following method to obtain an equation for the value of series resistor, r , which provides optimum linearity. Using the empirically derived

*Reference B1, page 55

**Reference B2, page 55

***Reference B3, page 55

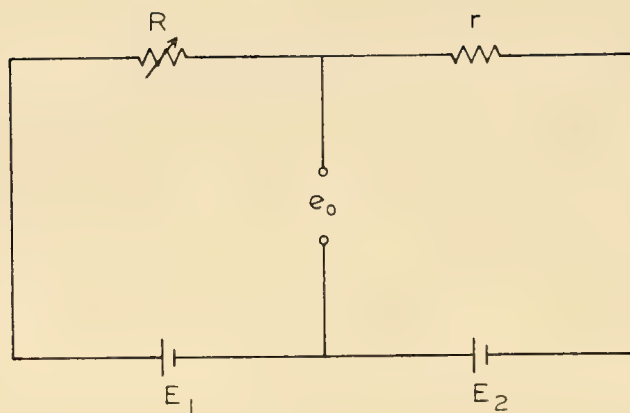


Figure B3. Asymmetric half-bridge

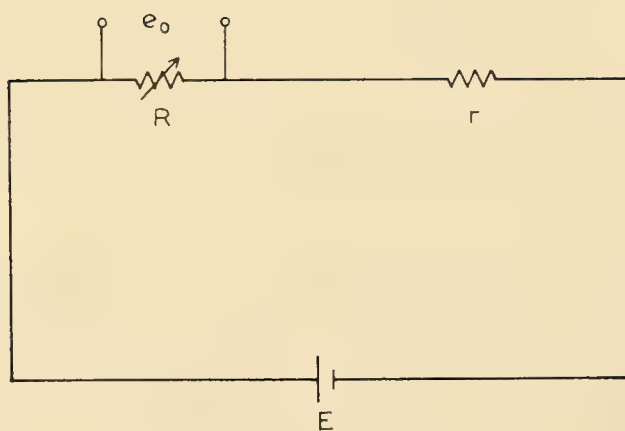


Figure B4. Beakley's bridge circuit

expression (eq. A1) $R = a \exp (b/T)$ for the resistance of a thermistor, Beakley wrote an expression, using Ohm's law, for the current through the bridge. He then expanded this into a Taylor's series. This series is linear if all derivatives with respect to T after the first are zero. He then showed that the second derivative term was the only significant one, and it was zero for the condition

$$r = \frac{b - 2T_o}{b + 2T_o} a \exp (b/T_o) \quad (B3)$$

where T_o is the absolute temperature at the midpoint of the range,

$b = \text{constant}$ (see below), and

$r = \text{optimum series resistor for linearity.}$

The constant, b , may be found by taking the derivative of the empirical expression (eq. A1) for thermistor resistance with respect to T

$$\frac{dR}{dT} = \frac{-b}{T^2} a \exp (b/T)$$

Solving for b ,

$$b = \frac{-T^2}{R} \frac{dR}{dT} \quad (B4)$$

Letting $a \exp (b/T_o)$ be R_o , eq. (B3) may be written,

$$r = \frac{b - 2T_o}{b + 2T_o} R_o \quad (B3a)$$

Beakley finds, among other things, an expression for the extent of the temperature range within which the departure from linearity is less than a specified value. This will be discussed later (see eq. B16a).

For work with thermistors, a good bridge circuit plus a method for obtaining optimum parameters which is easy to apply and involves a minimum of calculations while giving good results, is desirable. Since Burke's method and Kaufman's asymmetric half-bridge appeared to meet these requirements, they have been combined for our purpose. Results will be compared with those

of Beakley's method, which involves more calculations but can also be applied to the asymmetric half-bridge. Figures shown will also give more insight in showing the dependence of linearity on variations in the series resistor.

THEORY

In the following definitions, subscripts a , b , and c refer to low, medium, and high temperatures respectively (T_a , T_b , and T_c) within the desired temperature range. Thus

R_a, R_b , and R_c	= thermistor resistance
i_a, i_b , and i_c	= current through the bridge (figure B3)
e_{oa}, e_{ob} , and e_{oc}	= bridge voltage output
r	= linearizing series resistor
E_1 and E_2	= excitation potentials

An infinite load across the output was assumed and the excitation potentials were assumed to have zero impedance.

For linear voltage output as a function of temperature,

$$e_{oc} - e_{ob} = e_{ob} - e_{oa}$$

or

$$e_{oc} + e_{oa} - 2e_{ob} = 0 \quad (B5)$$

The outputs, e_{oa} , e_{ob} , and e_{oc} may be written,

$$e_{oa} = i_a r - E_2 \quad (B6)$$

$$e_{ob} = i_b r - E_2 \quad (B7)$$

$$e_{oc} = i_c r - E_2 \quad (B8)$$

Substituting these expressions into eq. (B5),

$$i_c + i_a - 2i_b = 0 \quad (B9)$$

The currents i_a , i_b and i_c may be written,

$$i_a = \frac{E_1 + E_2}{R_a + r} \quad (B10)$$

$$i_b = \frac{E_1 + E_2}{R_b + r} \quad (B11)$$

$$i_c = \frac{E_1 + E_2}{R_c + r} \quad (B12)$$

Substituting these expressions into eq. (B9),

$$\frac{E_1 + E_2}{R_a + r} + \frac{E_1 + E_2}{R_c + r} - \frac{2(E_1 + E_2)}{R_b + r} = 0 \quad (B13)$$

and since $E_1 + E_2 \neq 0$

$$\frac{1}{R_a + r} + \frac{1}{R_c + r} - \frac{2}{R_b + r} = 0 \quad (B14)$$

Solving eq. (B14) for r

$$r = \frac{2R_a R_c - R_c R_b - R_a R_b}{2R_b - R_c - R_a} \quad (B15)$$

This expression, therefore, defines the value of series resistor, r , for linearity in terms of the thermistor resistance in an asymmetric half-bridge circuit. This is the same expression as obtained by Burke for the optimum series resistor with a thermistor in a Wheatstone bridge.

From eq. (B15) it is seen that if the thermistor becomes more linear with temperature, the denominator approaches zero, and the value of r approaches infinity. Also, for a given thermistor, as the desired temperature range is decreased the value of r

increases. If the value of series resistor, r , becomes very large, the bridge becomes essentially a constant current device. These are the conditions when Kaufman* talks about large values of series resistors in half-bridges in connection with linear input devices such as platinum thermometers.

EXAMPLE

Table B1 gives the average resistance of five glass bead thermistors (VECO 32A1) at eleven different temperatures. These five thermistors have been compensated (see Appendix A) so that they all have very similar resistance-versus-temperature characteristics (the maximum deviation from the values stated below for any of the thermistors was only 1.3 ohms). The thermistors were calibrated in the NEL controlled temperature tank, which is capable of holding any desired temperature to within $\pm 0.002^\circ\text{C}$.**

TABLE B1. AVERAGE RESISTANCE OF GLASS
BEAD THERMISTORS.

<u>Temperature, $^\circ\text{C}$</u>	<u>Resistance, ohms</u>
9	3781.2
10	3640.0
11	3504.4
12	3374.0
13	3248.8
14	3128.9
15	3013.7
16	2903.3
17	2796.5
18	2694.2
19	2596.4

Let us compute the value of the linearizing series resistor, r , from eq. (B15). Let $R_a = 3640.0$ ohms, $R_b = 3128.9$ ohms, and $R_c = 2694.2$ ohms. Then $r = \underline{2687}$ ohms. Now obtain the value of r using the method of Beakley:***

*Reference B1, page 55

**Reference B4, page 55

***Reference B3, page 55

$$r = \frac{b - 2T_o}{b + 2T_o} R_o \quad (\text{B3a})$$

where R_o is the thermistor resistance at absolute temperature T_o , the midpoint of the desired temperature range, and

$$b = \frac{-T_o^2}{R_o} \frac{dR}{dT} \quad (\text{B4})$$

If we use the value from table B1 where $T_o = 14^\circ\text{C} = 287.16^\circ\text{K}$, at this temperature $\frac{dR}{dT} = 117.6$ ohms per $^\circ\text{C}$. Then from eq. (B3a) $r = \underline{2151}$ ohms.

DISCUSSION

In order to compare the degree of linearity obtained by using the two different values of series resistor, r , calculated above, a voltage output, e_o , was computed for both cases over the temperature range. For these computations the simplified circuit, figure B4, was used since in the derivation of eq. (B15) the value of the linearizing resistor is seen to be independent of E_1 and E_2 (see fig. B3). Using the values of thermistor resistance in table B1 and a value of linearizing series resistor, r , an excitation potential, E , was calculated by placing the condition that the current through the thermistor never exceed a given level (50×10^{-6} amperes). Having then fixed the circuit parameters (fig. B4) the output, e_o , was calculated for each value of thermistor resistance (1 degree centigrade intervals). These computed values of e_o as a function of temperature were then compared to a standard e_o which was a linear function of temperature in order to associate a quantitative measure of linearity with a given series resistor. The standard of linearity was chosen to be a straight line between the computed voltage outputs at 9 and 17°C . The reason for this choice will be discussed presently. The deviation of the computed output from this standard linear output was converted to a temperature deviation by referring to the slope of the

straight line in volts per °C. This deviation in °C for each series resistor is plotted over the temperature range 9 to 19°C in figure B5.

It can be seen from figure B5 that, for the conditions stated, 2670 ohms in series with the thermistor yielded the best linearity. This is shown more pointedly in figure B6 where the maximum departure from linearity is plotted as a function of the series resistor. As an example of how the data shown in figure B6 were obtained, refer to the curve labeled 2500 ohms in figure B5. An examination of this curve shows that the maximum departure from linearity over the full range is approximately $\pm 0.012^\circ\text{C}$. This then is the magnitude of linearity associated with 2500 ohms. Figure B6 shows pointedly the importance of finding the optimum value of the series resistor if extreme linearity is required. Note on this curve that the value given by the method of Beakley (eq. B3a) yielded poorer results than the value given by eq. (15); this may be due to the fact that for the particular thermistors considered (table B1), the empirical equation on which Beakley based his work did not hold well. There is some difficulty in obtaining the correct value of the constant, b , in the original empirical equation and eq. (B3a). However, there is also some uncertainty in applying the method of eq. (B15) because R_a , R_b , and R_c are only defined as the resistance at low, medium, and high temperatures. These points might be defined more precisely as the end points and the midpoint of the desired temperature range. This was done in the preceding example with good results.

In discussing linearity, care must be taken to define terminology. As was stated earlier, linearity in this paper refers to the departure of the function $e_0(T)$ from a straight line through $T=9$ and $T=17^\circ\text{C}$. This was not necessarily the best straight line fit for all cases considered, but merely a convenient standard for comparison.

Although, for our work, a 10°C range (9 to 19°C) was required, figure B7 may be useful in determining the degree of linearity that can be expected over different temperature ranges. Figure B7 is based on the following equation given by Beakley.*
The maximum error (departure from linearity) is given by:

*Reference B3, page 55

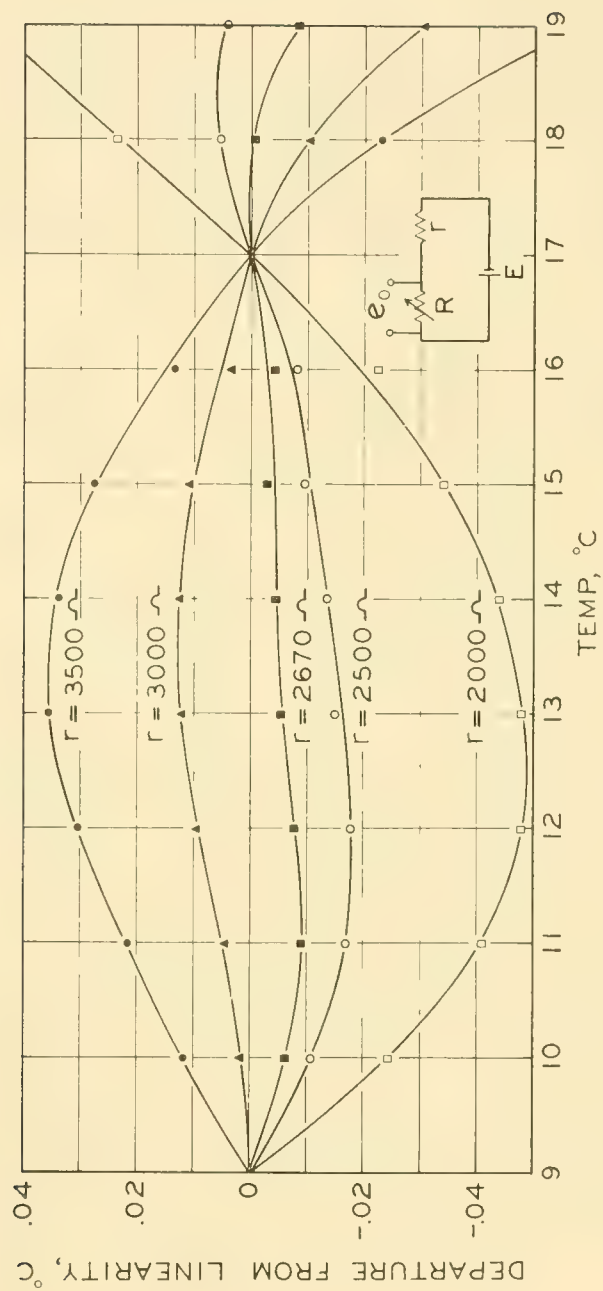


Figure B5. Departure from linearity vs. temperature for VECO 32A1 thermistors

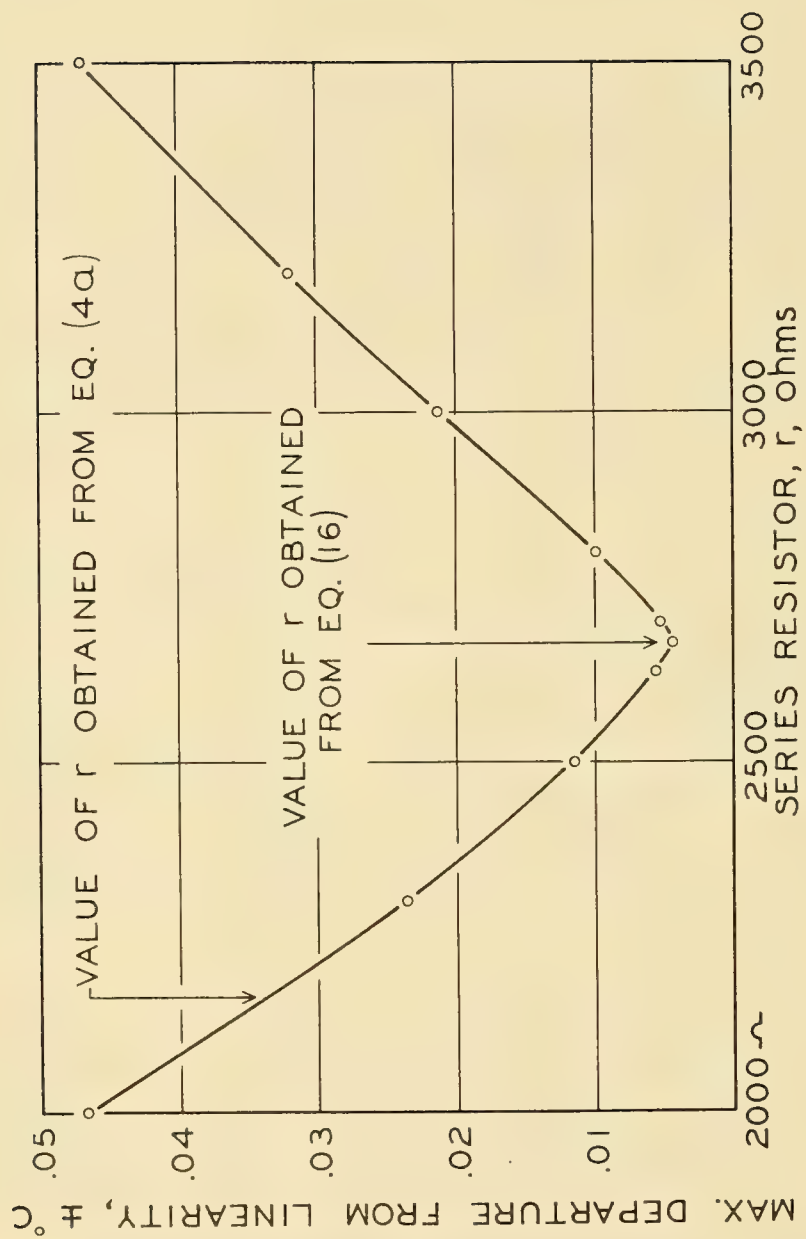


Figure B6. Maximum departure from linearity (10°C range) vs. series resistor for Veco 32A1 thermistors

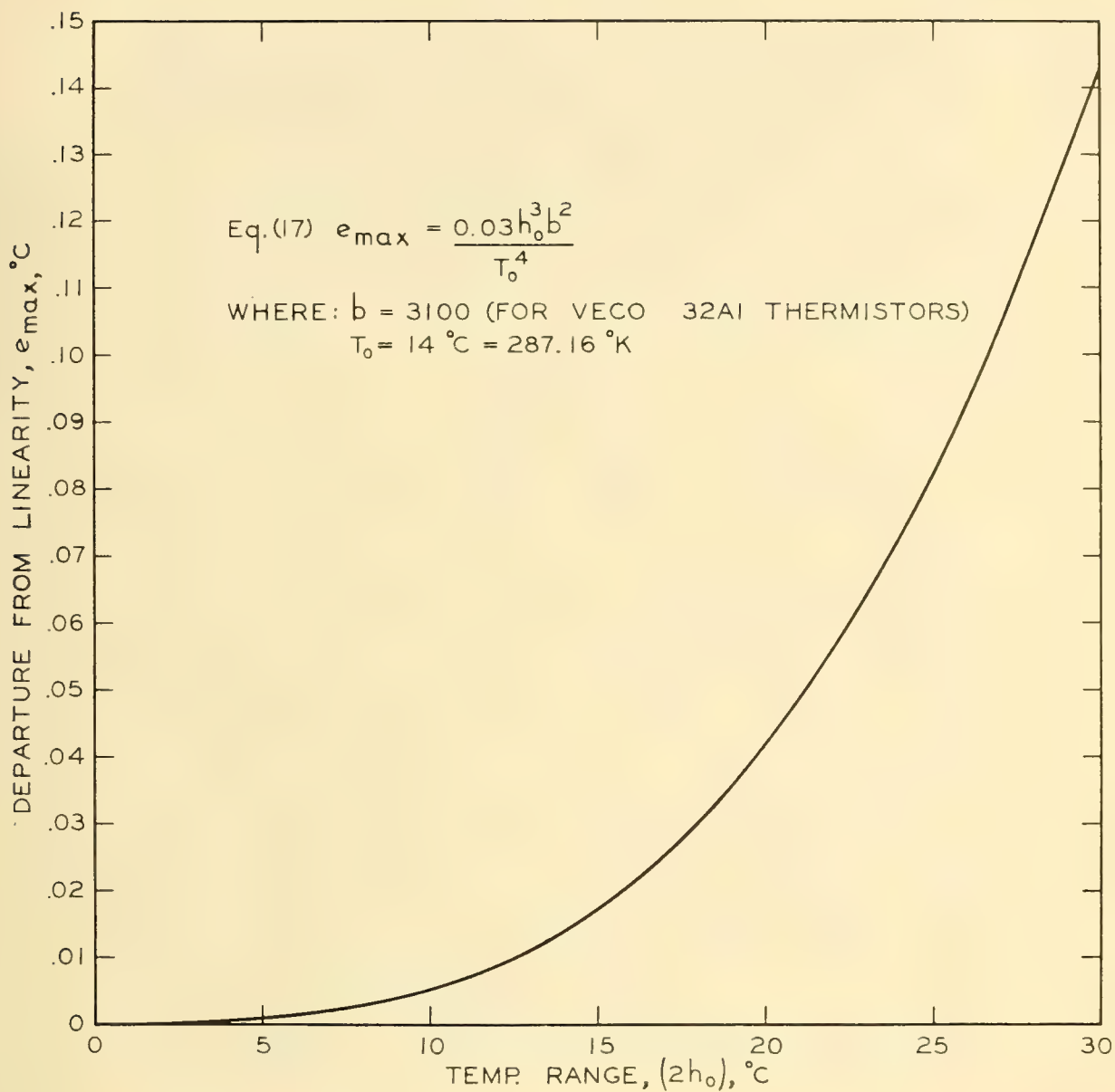


Figure B7. Maximum departure from linearity vs. temperature range

$$e_{\max} = 0.03 h_0^3 b^2 / T_0^4 \quad (\text{B16})$$

where T_0 is the absolute temperature at the midpoint of the range,
 $\pm h_0$ (in $^\circ\text{C}$) are the increments from T_0 to the end points of the

desired temperature range, and b is a constant for a particular type of thermistor as previously defined. This error function was obtained using the original empirical equation as the resistance of the thermistor in the circuit shown in figure B4. The predicted error (fig. B7) for a 10°C range is seen to be 0.004°C. This shows good agreement with our computations, since figure B6 gives an error of $\pm 0.0045^\circ\text{C}$ for the optimum parameters ($r = 2670$ ohms) over a 10°C range. If a 20°C range is required, figure B7 indicates that the maximum departure from linearity would be approximately 0.042°C.

CONCLUSIONS AND RECOMMENDATIONS

A thermistor temperature bridge, linear to ± 0.05 percent over a 10°C range, was obtained. This is the result of combining A. T. Burke's expression for the optimum series resistor with an asymmetric half-bridge circuit as described by Kaufman.* Both the method for linearizing and the circuit itself are readily adaptable to the fabrication of interchangeable, precision temperature probes. The results given here are based on computations, except for the original calibration of the thermistors.

If it is desired to design a linear temperature bridge using thermistors, it is recommended that reference should first be made to eq. (B16) which is plotted for a particular type of thermistor in figure 7. Here one can get an idea of the degree of linearity that can be expected once the temperature range is fixed. The value of the optimum linearizing series resistor can be obtained from eq. (B15). Reference may also be made to Beakley's method,** eq. (B3a).

*Reference B1, page 55

**Reference B3, page 55

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